

Should Retirees Insure Against Inflation or Just Worry About It?

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18 December 2007

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Abstract

The inflation rate for typical U.S. retirees is different from and mostly higher than the inflation rate for the population. The Consumer Price Index (CPI) has a lesser-known relative, the CPI-E (for the elderly) with sub-component weights based on the consumer expenditure survey for Americans above 62 years of age. Indeed, inflation doesn't seem to age well. This suggests that CPI-linked investments such as TIPS, I-Bonds and other real-return mutual funds, may not be the best hedge for individual retiree's increasing cost of living over an uncertain retirement horizon. To rigorously argue this point, our paper goes back to dynamic portfolio principles. We derive the optimal asset allocation between a CPI-linked bond fund and a generic investment fund, but for an aging retiree facing an exogenous liability stream that is imperfectly correlated to the real return fund. Our model trades-off the benefits of an imperfect insurance hedge against the risky potential for equity investment growth. Technically, our objective function is to minimize the probability of outliving one's wealth; which is not an unreasonable desire. Our numerical results indicate that although CPI-linked products are the bedrock of an optimal portfolio, the allocations can vary widely. Retirees who are concerned with maximizing lifetime sustainability – which is equivalent to minimizing retirement ruin – should expose themselves to nominal equity-based assets with potential for growth.

*The CPI as calculated may not be a conspiracy,
but it's definitely a con job foisted on an unwitting public
by government officials who choose to look the other way...*

Bill Gross, PIMCO Newsletter, October 2004

1 Introduction and Motivation

As the first baby boomer hits the age of 62 on January 1st 2008 and among other things becomes eligible for Social Security – and with over 70 million to follow during the next 20 years – the financial services industry is shifting its attention from wealth accumulation to generating a sustainable retirement income. Obviously, one of the main concerns in creating an appropriate income stream from a retirement nest egg such as a 401k or IRA, is the issue of inflation and price uncertainty over 25-35 potential years of retirement.

Against this backdrop, a number of financial commentators, including writers at the *Wall Street Journal* and the *New York Times*, have argued that retirees should allocate a substantial portion of their liquid retirement wealth to inflation-linked bonds such as TIPS and I-BONDS. These generate periodic income in real, as opposed to nominal, units.

For example, Zvi Bodie, in his book, *Risk Free Investing*, urges Americans to consider I-bonds and other CPI-linked products as the ultimate risk-free asset for retirement planning. This is more than just theory and academic advice. A number of large financial services companies, such as TIAA-CREF, PIMCO, Vanguard and Fidelity actively promote their inflation-linked mutual funds as a hedge against an increasing cost of living, especially for retirees. And, with the ongoing demise of traditional Defined Benefit (DB) pension plans, individuals who must rely on their 401k, IRA and other tax-sheltered accounts to generate a personal pension are starting to embrace the benefits of inflation-linked products.

Oddly enough, there seems to be very little discussion as to whether the Consumer Price Index (CPI-U) Urban – which is the underlying index of most inflation linked bonds – is an adequate measure of the actual inflation rate experienced by retirees. In fact, since the early 1980s the Bureau of Labor Statistics (BLS) has been calculating and tracking an experimental index called the CPI-E, for the elderly above the age of 62. This index weights

its expenditure sub-components based on the spending habits of the elderly, as opposed to the population as whole. See Amble and Stewart (1994) or Hobijn and Lagakos (2003) for more statistical and methodological details about this experimental index. The creation of this index was motivated by an ongoing movement within the U.S. house of Representatives to introduce a suitable inflation index for retirees that could be used to adjusted Social Security payments. Currently, Social Security payments in the U.S. are adjusted each year based on the previous year's inflation rate as measured by the CPI-W, which is a sub-component of the CPI-U.

We obviously don't want to get caught-up in the minutia of inflation measurement. Technically though, the CPI-W is for a representative household where more than one-half of the household's income must come from clerical or wage occupations and at least one of the household's earners must have been employed for at least 37 weeks during the previous 12 months. The CPI-W's population represents about 32 percent of the total U.S. population and is a subset, or part, of the CPI-U's (which stands for Urban) population. The CPI-U is used for computing the coupons and inflation-adjustments on Treasury Inflation Protection Securities (TIPS) as well as I-Bonds. From a practical perspective the CPI-W and the CPI-U are much closer to each other in composition, relative to the experimental CPI-E, which is why for most of the paper, we will refer to CPI vs. CPI-E.

Figure #1 exhibits the variation in behavior of the CPI-E versus the CPI-U during the last twenty years. Although both indices moved together and mostly upward over time, the CPI-E consistently exceeded the CPI (U,W) by more than 50 basis points per year. In some years the gap was even higher and the elderly household experienced an inflation rate that was a full percentage point above the general population.

Figure #1 Placed Here

The reason for this outperformance – or excessive inflation rate – is that retirees have different consumption and spending habits compared to the general population. They spend more of their disposable income on medical care and housing, and of course these prices have increased more than other subcomponents during the last 20 year period. Table #1 (all tables and charts are placed in the appendix) displays the relative importance and weights of the major expenditure categories in the respective indices. Note that medical care is

weighted 11% in the CPI-E but only 5% in the CPI-W, for example. Likewise, food is 13%% in the CPI-E but is a higher 16% in the CPI-W. Needless to say, any financial instrument whose returns are linked to the CPI create an imperfect hedge for a typical retiree's inflation rate.

Figure #2 is a scatter plot of the changes in the CPI-W versus changes in the CPI-E. If the correlation between shocks to the indices were 100%, then all point would obviously fall on a straight line. Notice that although the estimated R^2 is about 0.56 – which, using financial metrics is quite high – the relationship is not perfect. More importantly, we emphasize that each individual retiree will have their own unique weightings for consumer expenditures which might further reduce the correlations between their cost of living and the CPI. Thus, although a CPI-linked bond (or pension) for that matter is expected to increase over time as general prices increase – which is a good thing – it is important not to confuse an investment that increases over time with an investment that provides insurance and protection against unexpected shocks.

Table #1 Placed Here

Some authors – such as Jennings (2005), or Aziz, Katz and Prisman (2001) – have argued that as a result of these imperfect measures, CPI-linked bonds should be sliced or disaggregated into individual sub components so that consumers can reconstruct the bonds using their preferred weights. In this way a suitably-constructed and tailor-made bond fund would be generate payments and provide income based on a rate that is linked to the medical care component of the CPI. Retirees, and for that matter any consumer with a basket of consumption that differs from the general CPI, would be able to pick and choose their own inflation-linked bond fund that would hedge their own inflation-rate. If a retiree planned on only eating bananas somewhere on a desert island for the remainder of his entire life, he could purchase a bond fund that was linked to this (micro) sub component of the CPI.

And, while this idea is great in theory, consumers and current retirees who do not have access to – or are still waiting for – this engineering innovation must contend with a generic CPI-linked bond fund that is weakly correlated to their desired consumption's cost of living adjustments. The pragmatic and practical question they face is: *How much wealth should they allocate to an imperfect insurance policy, or are they better off taking their chances on*

a long-run risk premium that is expected to grow their portfolio over time?

Figure #2 Placed Here

This is the impetus for our paper. In the real-world absence of a perfect hedge, we gauge whether retirees are better-off investing in an asset class that is expected to grow over time – such as a diversified equity mutual fund – versus investing in an asset class that may or may not hedge a retiree’s cost of living. In this paper we develop a model which illustrates how the correlation, or lack thereof, between CPI-linked bond fund and a retiree’s personal inflation rate, impacts the demand and optimal allocation to this asset.

Now, many of these "lifecycle choice" models are built and calibrated assuming the consumer seeks to maximize their end-of-period utility of wealth or their ongoing utility of consumption under a variety of risk-aversion parameters and inter-temporal rates of substitutions. This approach to the dynamic asset allocation problem was initiated by Merton (1971) and has spawned hundreds of papers using these techniques. See Campbell and Viciera (2002) for an extensive literature review. A recent monograph by Ibbotson, et. al. (2007) takes the same utility-based approach as well. In related literature, Kothari and Shanken (2004) derive an optimal asset allocation with inflation-linked bonds, although not within the context of a retiree facing exogenous liabilities. However, we do use a number of their parameter estimates for our inputs, but differ in methodology and objective.

Our model is predicated on a retiree who is seeks to minimize the probability of lifetime ruin, assuming their expenditures are stochastic (but exogenous) in nominal terms. This framework has been adopted in a number of recent papers, see for example Browne (1995) or Young (2004), and has a number of intuitive benefits which resonate with individual investors and their financial advisors. A recent paper by Milevsky and Robinson (2005) derived a closed-form expression for the retirement ruin probability under a fixed spending strategy, and this probability is also at the heart of many Monte Carlo simulations embedded within retirement income software packages. This metric clearly resonates with some fraction of the wealth management community, which is why we have adopted it as framework for this analysis. In addition, minimizing ruin enables us to bypass measuring and calibrating the preference variables that would be required for a classical Mertonian approach. More importantly, we are able to directly measure the impact of not having a perfect hedge on the

probability of success and the affordability of a given projected liability, in addition to the demand for the imperfect asset. At the very least, we hopefully motivate readers to think more carefully about what is meant exactly by hedging and insuring against inflation risk.

From a methodological point of view, this paper extends the literature on ruin-minimizing optimal control to the case where there are three as opposed to just one, state variables, in an incomplete market. Obviously, the additional level of complexity eliminates the ability to derive analytic solutions and expressions for the optimal control, since we are left with a collection of PDEs. Our paper provides numerical results that help shed light on the optimal demand for CPI-linked bonds, by retiree's who are concerned with a simple objective function. Minimizing the probability of running out of money before running out of life.

Moreover – and from a practical perspective – using estimates of the actual correlation between various real-return bond funds offered by TIAA-CREF, PIMCO, Vanguard, etc, we find that a strong argument can be made for treating these funds as just another asset class as opposed to a unique product class that deserves a heavier portfolio weight. For example, when we regress monthly changes in the value of these funds against changes in the CPI-E, we arrive at correlation estimates on the order of 60% to 80%. Moreover, if one uses a different proxy for retiree inflation – for example one that includes a greater exposure to medical expenses – the correlation between shocks will be even lower.

Figure #3 Placed Here

Figure #3 displays the value of \$1 adjusted by the CPI-E (inflation) over the period 2002 to 2006. We compare against the value of \$1 invested in a hypothetical mutual fund that averages the net asset values of the four most popular inflation-linked bond funds (ILBFs) in the U.S. Notice that these values (obviously) deviate over time, both as a result of the mechanism for adjusting ILBFs and the "basis risk" with the CPI-E. This therefore leads us to wonder about the "optimal" allocation to ILBFs, given the rather unique nature of the retirement income problem.

The remainder of this paper is organized as follows. Section 2 develops our underlying optimization model. Section 3 provides numerical examples and displays some results and Section 4 concludes the paper. We have strived to minimize the mathematical derivations and moved all proofs to an appendix in order to focus attention on the main financial

implications.

2 The Underlying Model

Consistent with the ruin minimization literature – for example Browne (1999, 1995) – our model retiree starts with initial (nest egg) wealth denoted by $W_0 = w$, from which he/she spends at a rate of \$1 per annum, initially. Some practitioners refer to this as an *initial spending rate* of $1/w$. We assume that the retiree’s annual rate of desired consumption – measured in nominal terms – evolve according to the diffusion process:

$$dL_t = \pi L_t dt + \xi L_t dB_t^\pi, \quad L_0 = 1 \tag{1}$$

where the parameter π is the expected cost of living adjustment (COLA) which is unique to the retiree, σ_π is the volatility rate and B_t^π is the Brownian motion driving the uncertainty. For example, a retiree might expect his or her spending will increase by $\pi = 4\%$ each year, with a volatility of $\xi = 3\%$. Think of this as a personal inflation index/rate.

In our simple model, the retiree’s net-worth of $W_0 = w$ can only be allocated between two types of risky investment funds. The first is a low-risk CPI-linked bond fund that evolves according to the diffusion process:

$$dI_t = r I_t dt + \eta I_t dB_t^r, \quad I_0 = 1 \tag{2}$$

where r is the expected return – in nominal terms – and η is the volatility. In this case B_t^r denotes the Brownian motion driving the CPI-linked bond fund. It is important to emphasize that I_t does not represent the price of a particular CPI-linked bond, nor does it represent a coupon rate. Rather, one can best think of I_t as a unit value of a mutual fund that invests in a collection of CPI-linked bonds across various maturities, where all coupons and maturity payments are re-invested in the fund. An example of this type of a bond fund would include TIAA-CREF inflation-linked bond account, or PIMCO’s real-return bond fund, where the expectation is that the value of a given fund unit will keep-up with the CPI over time. The retiree would then liquidate or redeem units of the fund on a periodic basis and consume the proceeds. A typical parameter value could be an expected $r = 5\%$ investment return, which would consist of 2% real-return and perhaps 3% inflation, although

the exact decomposition is completely unnecessary for our analysis. The typical volatility η would be in the 8% range. These numbers are consistent with Ibbotson Associates (2006) estimates of the long-run volatility of a typical CPI-linked mutual fund, again in nominal terms.

The second investment alternative is an equity-based investment mutual fund that evolves according to the diffusion process:

$$dY_t = \mu Y_t dt + \sigma Y_t dB_t^y, \quad Y_0 = 1 \quad (3)$$

where μ is the expected return and $\sigma > 0$ is the volatility of the risky fund. Equation (3) is the workhorse of modern financial economics within the context of these models and goes back to Merton (1971). Typical parameter values are between 8% and 11% for the expected fund return μ and 10% to 20% for the fund volatility σ . Nothing new here.

Now, the correlation structure between the Brownian motions driving the uncertainty is as follows. First, the correlation between B_t^π (driving the retiree's personal inflation rate or COLA) and B_t^r (driving the CPI-linked bond fund), which is relatively high but not 100%, is denoted by $\rho_{\pi r}$. Likewise, the correlation between B_t^π and B_t^y (the mutual fund) is obviously lower than $\rho_{\pi r}$, and will be denoted by $\rho_{\pi y}$. Finally, the correlation between B_t^y and B_t^r is denoted by ρ_{ry} . Visually, the correlation structure is as follows:

$$\begin{array}{ccc|ccc}
 & \mathbf{L}_t & \mathbf{I}_t & \mathbf{Y}_t & & \\
 \hline
 & 1 & \rho_{\pi r} & \rho_{\pi y} & \mathbf{L}_t & \\
 \hline
 & \rho_{\pi r} & 1 & \rho_{ry} & \mathbf{I}_t & \\
 \hline
 & \rho_{\pi y} & \rho_{ry} & 1 & \mathbf{Y}_t & \\
 \hline
 \end{array} \quad , \quad \text{(C.Matrix)}$$

which – to be invertible – imposes some natural conditions on the relationship between the three available correlations, which will address later. In terms of calibration, Kothari and Shanken (2004) estimated the correlation between equity market returns and (hypothetical) CPI-linked funds returns are close to zero, so that $\rho_{ry} \approx 0$. For the most part, we will use this parameter estimate in our numerical examples. Likewise, the correlation $\rho_{\pi r}$ between the CPI-linked fund the retiree's COLA is a matter of debate, so we will use two extreme cases; the first being a low correlation of 40% and the second being a high correlation of 95%.

To anticipate our results, all else being equal we expect to see a higher allocation to the CPI-linked fund when it better hedges the COLA or personal inflation rate. This in fact will be confirmed. Finally, the correlation between the investment fund and the liabilities is partially constrained by the invertability of the correlation matrix, which we discuss later.

Putting this all together, the dynamics for the retirees investment portfolio will obey:

$$dW_t = \alpha_t W_t \frac{dI_t}{I_t} + (1 - \alpha_t) W_t \frac{dY_t}{Y_t} - L_t dt, \quad W_0 = w \quad (4)$$

where α_t is the fraction of the portfolio that is allocated to the CPI-linked bond fund and $(1 - \alpha_t)$ is the fraction allocated to the investment or equity-based mutual fund. Note that in this study L_t is defined as the consumption rate, i.e., consumption per unit of the time.

The retiree's mathematical problem is to minimize the probability of ruin over an retirement horizon $T = \infty$, but with a random time of death τ_d , under a particular law of mortality. The infinite horizon might sound odd in the context of a human life, but obviously the force of mortality will eventually kill the retiree well before $T = \infty$. so in some sense it is just a technicality. Also, the uncertainty in the length of human life can be hedged using pension annuities and our model can always be framed in terms of the choice between CPI-linked payout annuities and market-linked variable annuities¹.

Formally, the objective function can be stated as:

$$\psi(w, l, t) = \min_{0 \leq \alpha_t \leq 1} \Pr[\tau_0 < \tau_d | W_t = w, L_t = l, \tau_d > t], \quad (5)$$

where the additional constraint $\alpha_t \geq 0$ is imposed to preclude any short positions in the CPI-linked bond fund and $\alpha_t \leq 1$ is imposed to preclude any leveraged position in the equity fund. Our technical objective is to locate the optimal control $\alpha_t^*(w, t)$ as a function of all the drift and diffusion parameters $\mu, \sigma, \pi, \xi, r, \eta$ as well as the correlation parameters $\rho_{\pi r}, \rho_{\pi y}$ and ρ_{yr} . As we have stated earlier, we are particularly interested in how $\alpha_t^*(w, t)$ varies with respect to changes in the correlation $\rho_{\pi r}$ between the CPI and retiree's COLA or personal inflation rate.

Before we proceed to solve for the optimal control and the corresponding minimal ruin probability, we note the following qualitative observations. The qualitative interpretation of

¹See the paper by Charupat and Milevsky (2001) for the relationship and similarities between asset allocation and product allocation within the context of payout annuities.

$\xi = 0$ is that retiree's consumption liabilities will not fluctuate in nominal terms, although they can certainly fluctuate in CPI-adjusted terms. Likewise, the interpretation of $\eta = 0$ is that the CPI-linked bond fund grows deterministically over time at a rate of r , which means that there is no CPI risk in the system. Finally, when both $\xi = 0$ (i.e. no liability volatility) and $\eta = 0$ (no CPI risk) our problem boils down to minimizing the probability of ruin in a complete market, which has already been solved by Browne (1995, 1999) and Young (2004). In the static case, when α is fixed over the entire horizon, the ruin probability is available analytically, as derived by Milevsky and Robinson (2005).

Keep in mind that the absence of any risk-free asset in either nominal or personal-inflation adjusted terms, implies that the retiree can never reduce the ruin probability $\psi(w, t)$ to zero by immunizing (e.g. annuitizing) the consumption liability stream. The problem boils down to an economic trade-off between investing retirement wealth in an asset Y_t that is expected to earn $\mu > \pi$, and is greater than the projected increase in the retiree's cost of living, and investing in the relatively safe (and highly correlated) asset I_t , that is not expected to earn as much as Y_t , since $r < \mu$. Hence the title of this paper; should retirees insure against their inflation rate by allocating a substantial portion of their wealth to an imperfect hedge, or should they just "worry" about it, and instead invest more aggressively, perhaps hoping to grow/gamble their way out of the problem.

3 Numerical Solutions and Examples

We direct the interested reader to the appendix which contains the detailed "solution" of the relevant optimization problem as well as a description of the methodology we used to numerically solve the resulting Partial Differential Equation (PDE). The first risky asset Y_t , is a classical investment mutual fund that is expected to earn a nominal μ between 6% and 11% with a volatility of $\sigma = 20\%$. The second risky asset is the CPI-linked bond fund I_t , whose expected (nominal) return is $r = 5\%$ with a volatility of $\eta = 8\%$. Following the above-referenced work by Kothari and Shanken (2004), in most of our numerical examples presented we assume that the two risky assets (CPI-linked fund and equity/stock fund) are uncorrelated. Even though our methodology can readily handle any correlation between the two. In fact, one of the benefits of using two uncorrelated assets is that is deliberately

biases our results away from investing in the equity-based fund Y_t as a hedge against the retiree's liabilities.

The retiree starts with a consumption liability rate of $L_0 = \$1$ per year at retirement but at a varying level of initial wealth W_0 , allocated amongst Y_t and I_t . The retiree withdraws a stochastic $L_t, t > 0$ per year. We assume that $0 \leq \alpha_t \leq 1$, and thus prohibit leverage and short-sales. We assume that L_t increases by $\pi = 4\%$ per annum, under two separate cases; volatility of $\xi = 3\%$ (high) and 1% (low). We assume that the CPI-linked fund has a higher expected return r , relative to the liabilities $\mu = 4\%$ and with greater volatility $\eta = 8\%$. Finally, we are interested in the minimal probability of ruin assuming the correlation between liabilities L_t and bond funds returns I_t is $\rho_{\pi r} = 95\%$ (high) and 40% (low). We are interested in how the decreasing return of the equity-based asset will force a greater allocation to the bond-fund. We are also interested in the minimal ruin probability as a function of the correlation between the CPI-based bond fund and the consumption liability $\rho_{\pi r}$. The value of the remaining correlation coefficient is taken to be $\rho_{\pi y} = 0.25$ which is chosen so that the positivity of the correlation matrix is guaranteed.

All the computations are carried out with an initial retirement age of $y = 65$ and a time horizon of $T = 60$ (a maximum age of 125). We have experimented with a longer time horizon and the results are virtually identical, since obviously the retiree is dead well before $T = 60$. We used the so-called Gompertz mortality rate in the numerical results, which implies that the actuarial force of mortality satisfies the following equation:

$$\lambda_y = \lambda_0 + \frac{1}{b} \exp\left(\frac{y - m}{b}\right) \quad (6)$$

The hazard/mortality rates becomes sufficiently large for a large T , so the survival rate is negligible after 125 years of life. For the result we displayed, we used the set of parameter values: $m = 86.3$, $b = 9.5$ and $\lambda_0 = 0.003$. Also, to carry out the computations, we have replaced the infinite domain $0 \leq x < \infty$ by a finite one $0 \leq x \leq x_{max}$. We found that as long as x_{max} is sufficiently large, the effect on the solution is small. We have chosen $x_{max} = 200$. This is equivalent to assuming that the ruin probability is zero when the wealth to liability ratio is 200. A total of 1000 grid points are used for the state variable $x = w/l$ and 240 time steps in time. We also experimented with larger numbers of grid points in x and y and almost identical results were obtained. Finally, we have also compared the solutions

obtained by explicit and implicit methods. Again, the results are virtually the same.

Table #2a Placed Here

In Table # 2a we present the computed values of the minimum ruin probability and the asset allocation percentage α^* for a high correlation $\rho_{\pi r} = 95\%$ between the COLA and the CPI-linked fund, together with a high volatility for the consumption liability $\xi = 3\%$. We are interested in the interaction of the assumed equity return μ and the initial spending rate $1/w$.

Here is how to interpret the results. A 65 year-old retiree with initial investment wealth (nest egg) of \$1,000,000 wants to spend/consume \$40,000 per year adjusted for personal inflation. In the language of our model, this is a desired spending rate of 4% and is equivalent to an initial wealth to consumption ratio of $W_0 = 25$. The retiree estimates that this liability L_t will increase on average by $\pi = 4\%$ per year (using continuous compounding) with a volatility of $\xi = 3\%$. The retiree has a choice of investing wealth between an CPI-linked mutual fund that is expected to earn $r = 5\%$ per year with a volatility of $\eta = 8\%$, and a riskier equity based fund. The fund has a volatility of $\sigma = 20\%$, and the expected return is denoted by μ . Table #2 displays the optimal allocation to the CPI-linked fund depending on the retiree's subjective estimate of the expected return μ . For example, if the equity fund is expected to earn a mere $\mu = 6\%$, which is 100 basis point more than the CPI-linked fund, the initial optimal allocation to the CPI-linked fund is 72.6%. The other 27.4% is allocated to the equity fund. Obviously, since the expected return μ from the equity fund is low the majority of the nest egg should rationally be placed in the CPI-linked fund. In this case, under the optimal strategy, the probability of ruin is 24.4%. This is the lowest possible ruin probability – from the set of all possible dynamic investment strategies – given the specified parameters. Now, if the retiree expects the return on the equity fund to be $\mu = 11\%$, which is 500 basis points higher than the previous example, then the optimal allocation the CPI-linked fund is a (much) lower 56%. This means that 64% should be allocated to equity, versus 27.4%. Naturally, if one is more bullish about equity, the allocation is greater. In this case, the minimal ruin probability is a mere 6.5% as opposed to 24.4%.

Table #2b Placed Here

All of the numbers in Table #2a assume that the correlation between the (shocks to) retirement liabilities L_t and the CPI-linked fund returns are quite high, at $\rho_{\pi r} = 95\%$. To contrast this case, in Table # 2b we display the results of the minimum ruin probability P^* and optimal asset allocation percentage α^* for a lower correlation of $\rho_{\pi r} = 40\%$ between the personal inflation rate shocks and the returns to the CPI-linked fund. We still assume the (high) volatility of personal inflation, $\xi = 3\%$. In this case, notice that for the same earlier-mentioned $W_0 = 25$, which is a spending rate of 4%, the optimal allocations to the CPI-linked fund are (generally, although not always) lower. The minimal ruin probabilities P^* are uniformly higher in Table #2b compare to Table #2a. The intuition is simple. If the CPI-linked fund is a "worse" hedge for the liabilities, the probability of ruin is higher, even under the most efficient or optimal investment strategy.

Table #3 Placed Here

In Table #3 we present a different perspective on the results, by comparing the allocations as a function of perceived equity-market volatility. For example, a retiree who expects equities to earn $\mu = 10\%$ and market volatility to be on the order of $\sigma = 20\%$, should allocate 72% of his nest egg to CPI-linked bonds. However, if the volatility (can be controlled) and is reduced to a mere $\sigma = 10\%$ annualized, then the optimal allocation to the CPI-linked fund falls to 48%. And, these results are when the correlation between personal inflation and returns to the CPI-linked fund are high at $\rho_{\pi r} = 95\%$. When the correlation is much weaker at $\rho_{\pi r} = 25\%$, the optimal allocations to the CPI-linked fund drop to between 32% and 65%.

Finally, in order to provide insights on how a dynamic (versus static) asset allocation strategy helps to lower the ruin probability, we have plotted several key quantities in Figure #4. The underlying parameter values are the same as those in Table #2a with an expected equity return of $\mu = 8\%$. First, the minimum ruin probability P^* as well as the ruin probabilities assuming a 100% allocation to the CPI-linked bond fund ($\alpha = 1$) or 100% allocation to the equity-based fund ($\alpha = 0$) are presented in Figure #4a. Notice how the optimal strategy results in a ruin probability that is lower than either extreme allocations. Then, the variation of the minimum ruin probability with age is plotted in Figure #4b and the dynamics of the optimal asset allocation ratio α^* at different ages is plotted in Figure

#4c. To gain some intuition for Figures #4b and #4c one should focus on the x-axis point where w/l is 20 units. This represents an individual with assets worth 20 times their current level of consumption. At the age of 80, this is a very fortunate situation since the ruin probability is less than 5% in Figure #4b. However, at the age of 65, this wealth to liability ratio induces a ruin probability of approximately 20%. Obviously, as the wealth to liability ratio increases, the entire vector of ruin probabilities decline, and reach zero at approximately 50-to-1.

Figure #4a, #4b, #4c Placed Here

Notice in Figure #4c, the optimal allocation to the CPI-linked fund, which is represented in the y-axis, depends on age but asymptotes to about 80% when the wealth ratios are very high. At lower levels of wealth, the allocation to the CPI-linked fund is zero, especially at younger ages.

One of the main objectives of this study is to investigate the "correlation impact". To provide a graphical illustration we have plotted α^* (asset allocation) and ψ (ruin probability) against the wealth to liability ratio w/l for three different correlation scenarios: $\rho_{\pi r} = 0, 0.4, 0.95$. The results are displayed in Figure #5. This allows us to directly observe (only) the impact of the correlation between the personal inflation rate and the returns to the CPI-linked fund. One of the more noticeable results is the reversal of the asset allocation strategy – from safety to gambling – at a critical value of w/l . This is typical of all ruin minimization models, see Browne (1999) for example, where at some point the retiree decided to gamble in order to make ends meet.

Figure #5a, #5b Placed Here

In sum, we obviously don't expect the normative model we developed here to drive actual asset allocation recommendations at retirement, especially given the rather restricted nature of the assumed price dynamics (Geometric Brownian Motion) and investment choices. Furthermore, our selection of a *minimum ruin* criteria, as opposed to a more common but abstract *utility maximization* criteria is meant to flush-out the following point. Even the most risk averse preferences – i.e. "my main goal is to never run out of money" – do not necessarily induce a high allocation to the so-called safest asset, *when the hedging and insurance isn't*

perfect. To some readers this result might be obvious in an incomplete market, but we believe that it is worth emphasizing and illustrating in the context of inflation and the cost of living in retirement. There is no guarantee that a CPI-linked fund will hedge personal liabilities. In our opinion, it should be treated as just another asset class in the investment pool, but with a unique set of risk, return and correlation parameters.

4 Conclusion

Few people realize that the inflation rate for U.S. retirees is different from and mostly higher than the inflation rate for the aggregate population. The widely referenced and often criticized Consumer Price Index (CPI) has a lesser-known relative labeled the CPI-Elderly (CPI-E), with component weights based on the consumer expenditure survey for Americans above the age of 62. Among other things, this implies that CPI-linked investments such as TIPS, I-Bonds and other real-return mutual funds, might not be a good hedge for retiree's increasing cost of living over an uncertain retirement horizon.

To expand on this idea in a more rigorous manner, this paper derives the optimal asset allocation between a CPI-linked bond fund and a generic equity mutual fund, for a retiree facing a liability stream that is *imperfectly correlated* to returns on the CPI-linked fund. Our model shines a direct light on the risk return trade-off between the benefits of imperfect insurance versus the potential for investment growth. Our objective function is to minimize the probability of outliving one's money. Our analysis confirms that CPI-linked products should play a substantial role in the optimal retirement portfolio. However, we also sound a note of caution. And, when one considers the overall criticism that has been leveled against the CPI – such as the quote by Bill Gross at the opening of this paper – real return funds may not hedge against an uncertain inflation rate during retirement.

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APPENDIX: Derivation and Solution of the HJB

In this appendix, we derive the Hamilton-Jacobi-Bellman (HJB) equation satisfied by $\psi(w, l, t)$ and discuss methodologies for solving the HJB.

A.1 Derivation of the HJB

The derivation is based on the method in Björk (1998, chapter 14) and described by Browne (1999) or Young (2004). Assume an individual of age y at time zero. At any time t the individual aged $(y + t)$ continues the optimal asset allocation strategy within the range $0 \leq \alpha \leq 1$ if they survive until $t + h$. On the other hand, if $(y + t)$ dies before time $t + h$, then the probability of ruin is zero. Let $\psi(w, l, t)$ denote the minimum ruin probability, i.e.,

$$\psi(w, l, t) = \min_{\alpha_t} Pr[\tau_0 < \tau_d | W_t = w, L_t = l, \tau_d > t]. \quad (\text{A.1})$$

Therefore,

$$\psi(w, l, t) \leq E[\psi(W_{t+h}, L_{t+h}, t+h) | W_t = w, L_t = l] ({}_h p_{y+t}) \quad (\text{A.2})$$

$$+ E[0 | W_t = w, L_t = l] ({}_h q_{y+t}) \quad (\text{A.3})$$

where the equality holds if and only if the optimal strategy is used. Here the symbols $({}_h p_{y+t})$ and $({}_h q_{y+t}) := 1 - ({}_h p_{y+t})$ are standard actuarial notation for the probabilities of surviving and dying between the time period h for an individual aged $(y + t)$, given by

$${}_h p_y = e^{-\int_0^h \lambda_{y+s} ds},$$

where λ_t is the instantaneous hazard rate of the force of mortality. Now, let \mathcal{A} denote the operator defined as follows

$$\begin{aligned} \mathcal{A}f(w, l, t) &= f_t + [\alpha r w + (1 - \alpha)\mu w - l] f_w \\ &\quad + \frac{1}{2} \{ (\alpha\eta)^2 + [(1 - \alpha)\sigma]^2 + 2\alpha(1 - \alpha)\rho_{ry}\eta\sigma \} w^2 f_{ww} \\ &\quad + \pi l f_l + \frac{1}{2} (\xi l)^2 f_{ll} + [\alpha\rho_{\pi r}\eta + (1 - \alpha)\rho_{\pi y}\sigma] \xi w l f_{wl}. \end{aligned} \quad (\text{A.4})$$

Using Ito's lemma on the function ψ , we have

$$\begin{aligned}\psi(W_{t+h}, L_{t+h}, t+h) &= \psi(w, l, t) + \int_t^{t+h} \mathcal{A}\psi(w_s, l_s, s) ds \\ &+ \int_t^{t+h} \psi_w(w_s, l_s, s) [\alpha \eta w_s dB_s^r + (1-\alpha) \sigma w_s dB_s^y] \\ &+ \int_t^{t+h} \psi_l(w_s, l_s, s) \xi l dB_s^\pi.\end{aligned}\tag{A.5}$$

Combining equation (A.2), (A.4) and (A.5), we obtain

$$\psi(w, l, t) \leq {}_h p_{y+1} E^{w, l, t} \left[\psi(w, l, t) + \int_t^{t+h} \mathcal{A}\psi(W_s, L_s, s) ds \right],\tag{A.6}$$

or

$$\psi(w, l, t) {}_h q_{y+t} \leq {}_h p_{y+1} E^{w, l, t} \left[\int_t^{t+h} \mathcal{A}\psi(W_s, L_s, s) ds \right].\tag{A.7}$$

Taking the limit of $h \rightarrow 0$ and using $\lim_{h \rightarrow 0} h^{-1} {}_h q_{y+t} = \lambda_{y+t}$, we obtain

$$\psi(w, l, t) \lambda_{y+t} \leq \mathcal{A}\psi(w, l, t)\tag{A.8}$$

and the equality holds when the optimal strategy is used, which is either given by the first order condition or the boundary values $\alpha = 0$ or 1 . Thus we obtain the HJB equation

$$\psi(w, l, t) \lambda_{y+t} = \min_{0 \leq \alpha \leq 1} \{ \mathcal{A}\psi(w, l, t) \}.\tag{A.9}$$

The boundary conditions are

$$\psi(\infty, l, t) = 0, \quad \psi(0, l, t) = 1.\tag{A.10}$$

Assuming an infinite time horizon, the terminal condition is

$$\psi(w, l, \infty) = 0.\tag{A.11}$$

When the mortality rate λ_{y+t} is a constant, the infinite time horizon case leads to a time-independent version of the (A.9) by dropping the term ψ_t . However, this is not the case (for realistic biological mortality) where the variable λ_{y+t} is time dependent and the solution ψ is therefore also time-dependent. In this case, time is equivalent to age, and we can rewrite the Hamilton-Jacobi-Bellman equation (A.9) using age y as the time variable

$$\lambda_y \psi = \psi_y + \min_{0 \leq \alpha \leq 1} \mathcal{H},\tag{A.12}$$

where

$$\begin{aligned}
\mathcal{H} &:= [\alpha r w + (1 - \alpha)\mu w - l] \psi_w + \pi l \psi_l \\
&+ \frac{1}{2} \{ (\alpha \eta)^2 + [(1 - \alpha)\sigma]^2 + 2\alpha(1 - \alpha)\rho_{ry}\eta\sigma \} w^2 \psi_{ww} \\
&+ \frac{1}{2} (\xi l)^2 \psi_{ll} + [\alpha \rho_{\pi r} \eta + (1 - \alpha)\rho_{\pi y} \sigma] \xi w l \psi_{wl}.
\end{aligned} \tag{A.13}$$

The boundary conditions are still:

$$\psi(\infty, l, y) = 0, \quad \psi(0, l, y) = 1, \quad \psi(w, l, \infty) = 0. \tag{A.14}$$

A.2 Solution procedure

By introducing a similarity variable $x = w/l$, which is the ratio of wealth to the liabilities, it can be shown that the now two dimensional equation $\psi(x, y)$ satisfies

$$\lambda_y \psi = \psi_y + \mathcal{H}_*, \tag{A.15}$$

where

$$\begin{aligned}
\mathcal{H}_* &:= \frac{1}{2} \{ [\alpha_*^2 \eta^2 + (1 - \alpha_*)^2 \sigma^2 + 2\alpha_*(1 - \alpha_*)\rho_{ry}\eta\sigma] + \xi^2 - 2[\alpha_*\rho_{\pi r}\eta + (1 - \alpha_*)\rho_{\pi y}\sigma] \xi \} x^2 \psi_{xx} \\
&+ \{ [\alpha_* r x + (1 - \alpha_*)\mu x - 1 - \pi x] + \xi^2 x - [\alpha_*\rho_{\pi r}\eta + (1 - \alpha_*)\rho_{\pi y}\sigma] \xi x \} \psi_x,
\end{aligned} \tag{A.16}$$

and the value of α_* is either the boundary value (0 or 1) or given by the first order condition

$$\alpha_* = -\frac{(\rho_{\pi y}\sigma - \rho_{\pi r}\eta)\xi + r - \mu}{\eta^2 + \sigma^2 - 2\rho_{ry}\eta\sigma} \frac{\psi_x}{x\psi_{xx}} - \frac{\sigma(\rho_{ry}\eta - \sigma) + (\rho_{\pi y}\sigma - \rho_{\pi r}\eta)\xi}{\eta^2 + \sigma^2 - 2\rho_{ry}\eta\sigma}. \tag{A.17}$$

(Note that in the appendix we used α_* instead of α^* for typographical reasons.) There are two ways one can use to actually derive equation (A.15). By treating $X_t = W_t/L_t$ as the main stochastic variable and applying the Ito's lemma

$$\begin{aligned}
dX_t &= \{ [\alpha_* r + (1 - \alpha_*)\mu X_t - 1 - \pi X_t] + \xi^2 X_t - [\alpha_*\rho_{\pi r}\eta + (1 - \alpha_*)\rho_{\pi y}\sigma] \xi X_t \} dt \\
&- \xi X_t dB_t^\pi + \alpha \eta X_t dB_t^r + (1 - \alpha)\sigma X_t dB_t^y,
\end{aligned} \tag{A.18}$$

we can follow the same procedure outlined in Section A.1, combined with the first order condition. That will lead to (A.15). Alternative, one can simply apply the chain-rule to

(A.13) and then use the first order condition (A.17). The derivations are straightforward in both cases and the details are omitted in this paper. The boundary conditions are

$$\psi(0, y) = 1, \quad \psi(\infty, y) = 0, \quad \psi(x, \infty) = 0. \quad (\text{A.19})$$

To solve equation (A.15) numerically we need to truncate the domain in x as well as y and replace the boundary conditions in (A.19) by;

$$\psi(0, y) = 1, \quad \psi(x_{max}, y) = 0, \quad \psi(x, y_{max}) = 0 \quad (\text{A.20})$$

where x_{max} and y_{max} are relatively large numbers. The computational results presented in this paper are obtained by choosing $x_{max} = 200$ and $y_{max} = 125$, based on the observation that further increasing the size of the domain produces almost identical solutions.

There are two possible ways to solve (A.15), one by discretising the time derivative ψ_y *explicitly* while the other by using an *implicitly* method. In both cases, we discretize y as $y_n = y_0 + n\delta y$ for $n = 0, 1, \dots, N$, where $\delta y = (y_{max} - y_0)/N$, y_0 is the current age of the retiree.

A.2.1 Explicit method

Let $\psi^{(n)}$ denote ψ at time (age) y_n . The HJB (A.12) is approximated by

$$\begin{aligned} \psi^{(n+1)} &= \psi^{(n)} + \delta y F(\psi^{(n)}), & (\text{A.21}) \\ F(\psi^{(n)}) &= -\lambda_y \psi^{(n)} \\ &+ \frac{1}{2} \left\{ [\alpha_*^2 \eta^2 + (1 - \alpha_*)^2 \sigma^2 + 2\alpha_*(1 - \alpha_*)\rho_{ry}\eta\sigma] \right. \\ &\quad \left. + \xi^2 - 2[\alpha_*\rho_{\pi r}\eta + (1 - \alpha_*)\rho_{\pi y}\sigma] \xi \right\} x^2 \psi_{xx}^{(n)} \\ &+ \left\{ [\alpha_* r x + (1 - \alpha_*)\mu x - 1 - \pi x] + \xi^2 x \right. \\ &\quad \left. - [\alpha_*\rho_{\pi r}\eta + (1 - \alpha_*)\rho_{\pi y}\sigma] \xi x \right\} \psi_x^{(n)}. & (\text{A.22}) \end{aligned}$$

Let $\psi^{(n,*)}(x)$ denotes the solution obtained using (A.22) and the first order condition (A.17), $\psi^{(n,0)}(x)$ and $\psi^{(n,1)}(x)$ the solution of (A.22) using $\alpha_* = 0$ or $\alpha_* = 1$. We choose the final solution as

$$\psi^{(n)}(x) = \min \left\{ \psi^{(n,*)}(x), \psi^{(n,0)}(x), \psi^{(n,1)}(x) \right\}.$$

This method is conceptually simple and easy to implement. However, it is not efficient as small δy has to be used due to stability constraint associated with the explicit method.

A.2.2 Implicit method

Alternatively, we can use an implicit method by approximating the equation (A.12) as

$$\psi^{(n+1)} = \psi^{(n)} + \delta y F(\psi^{(n+1)}), \quad (\text{A.23})$$

$$\begin{aligned} F(\psi^{(n+1)}) = & -\lambda_y \psi^{(n+1)} \\ & + \frac{1}{2} \{ [\alpha_*^2 \eta^2 + (1 - \alpha_*)^2 \sigma^2 + 2\alpha_*(1 - \alpha_*)\rho_{ry}\eta\sigma] \\ & + \xi^2 - 2[\alpha_*\rho_{\pi r}\eta + (1 - \alpha_*)\rho_{\pi y}\sigma] \xi \} x^2 \psi_{xx}^{(n+1)} \\ & + \{ [\alpha_* r x + (1 - \alpha_*)\mu x - 1 - \pi x] + \xi^2 x \\ & - [\alpha_*\rho_{\pi r}\eta + (1 - \alpha_*)\rho_{\pi y}\sigma] \xi x \} \psi_x^{(n+1)}. \end{aligned} \quad (\text{A.24})$$

The simple approach used in the explicit method is not applicable here as we have to determine α_* before computing the probability $\psi^{(n+1)}$. In fact, we need to use the second order condition $\mathcal{H}_{\alpha\alpha} > 0$ at the minimum value. It can be verified that

$$\mathcal{H}_{\alpha\alpha} = (\eta^2 + \sigma^2 - 2\rho_{ry}\eta\sigma)^2 x^2 \psi_{xx}.$$

Note that ψ is not convex and $\mathcal{H}_{\alpha\alpha}$ can change signs. Therefore, we need both the first and second order conditions to determine the value of α_* .

- If $\mathcal{H}_{\alpha\alpha} > 0$ and α_* given by (A.17) satisfies $0 \leq \alpha_* \leq 1$, then that is the value for α_* ;
- If $\mathcal{H}_{\alpha\alpha} > 0$ and α_* given by (A.17) satisfies $\alpha_* > 1$, then $\alpha_* = 1$;
- If $\mathcal{H}_{\alpha\alpha} > 0$ and α_* given by (A.17) satisfies $\alpha_* < 0$, then $\alpha_* = 0$;
- If $\mathcal{H}_{\alpha\alpha} \leq 0$, we need to compare \mathcal{H}_α at $\alpha = 0$ and $\alpha = 1$ and choose the α value which gives the smaller \mathcal{H}_α .

By comparison, this implicit method is much more efficient than the explicit one presented earlier since relatively large δy can be used for the computation.

Table #1

Table #1: Components of the CPI: Wage-earners (W) vs. Elderly (E)		
Spending	% of CPI-W	% of CPI-E
Apparel	4%	2%
Education & Communication	6%	3%
Food & Beverage	16%	13%
Housing	40%	48%
Medical Care	5%	11%
Recreation	5%	5%
Transportation	20%	14%
Other Goods & Services	4%	4%
Total	100%	100%

These weights are from December 2006. Note that weights vary over time based on the consumer expenditure survey conducted by the Bureau of Labour Statistics.

Table #2a: OPTIMAL ALLOCATION TO CPI-LINKED FUND								
High ($\rho_{\pi r} = 95\%$) Correlation with Personal Inflation Rate								
High ($\xi = 3\%$) Volatility of Personal Inflation Rate; Expected $\pi = 4\%$								
Expected	$W_0 = 18$		$W_0 = 20$		$W_0 = 22$		$W_0 = 25$	
Investment	Spend = 5.5%		Spend = 5.0%		Spend = 4.54%		Spend = 4.0%	
Fund Return	α^*	P^*	α^*	P^*	α^*	P^*	α^*	P^*
$\mu = 6\%$	0	0.478	0.300	0.407	0.541	0.339	0.726	0.244
$\mu = 7\%$	0.076	0.430	0.306	0.359	0.477	0.293	0.642	0.205
$\mu = 8\%$	0.127	0.378	0.312	0.307	0.453	0.244	0.599	0.165
$\mu = 9\%$	0.165	0.324	0.323	0.256	0.445	0.197	0.576	0.126
$\mu = 10\%$	0.201	0.271	0.339	0.206	0.447	0.153	0.564	0.093
$\mu = 11\%$	0.233	0.219	0.357	0.160	0.454	0.114	0.560	0.065
Note: Assets that are not invested in the CPI-linked Bond Fund								
are allocated to Equity Fund with mean return μ , and volatility $\sigma = 20\%$								
The expected return from CPI-linked Fund is $r = 5\%$, with volatility $\eta = 8\%$.								
Correlation between CPI-linked fund and Equity Fund return is $\rho_{ry} = 0\%$.								
Correlation between personal inflation and Equity Fund return is $\rho_{\pi y} = 25\%$.								

Note that there is a constraint on the third correlation figure, $\rho_{\pi y}$ needed for the positivity of the covariance matrix. For $\rho_{ry} = 0$ and $\rho_{\pi r} = 0.95$, we find that $\rho_{\pi y} < 31.225\%$. In the computations, we have chosen $\rho_{\pi y} = 25\%$. All of these numbers are consistent with – although not identical to – the Ibbotson Associates (2006) estimates for the behavior of TIPS-based funds which invest in long maturity CPI-linked bonds. The underlying model assumes the retiree starts with W_0 in liquid assets and spends $L_0 = 1$ per year, which increases by an expected $\pi = 4\%$ with a volatility of ξ per year. The column headed by α^* denotes the optimal allocation and the column headed by P^* denotes the minimum ruin probability.

Table #2b: OPTIMAL ALLOCATION TO CPI-LINKED FUND								
Low ($\rho_{\pi r} = 40\%$) Correlation with Personal Inflation Rate								
High ($\xi = 3\%$) Volatility of Personal Inflation Rate; Expected $\pi = 4\%$								
Expected	$W_0 = 18$		$W_0 = 20$		$W_0 = 22$		$W_0 = 25$	
Investment	Spend = 5.5%		Spend = 5.0%		Spend = 4.54%		Spend = 4.0%	
Fund Return	α^*	P^*	α^*	P^*	α^*	P^*	α^*	P^*
$\mu = 6\%$	0.089	0.479	0.435	0.408	0.624	0.340	0.735	0.249
$\mu = 7\%$	0.143	0.432	0.366	0.361	0.517	0.296	0.645	0.212
$\mu = 8\%$	0.164	0.381	0.341	0.311	0.468	0.249	0.590	0.172
$\mu = 9\%$	0.185	0.328	0.334	0.260	0.444	0.203	0.556	0.135
$\mu = 10\%$	0.206	0.275	0.337	0.211	0.434	0.159	0.535	0.101
$\mu = 11\%$	0.229	0.223	0.344	0.166	0.431	0.120	0.523	0.072
Note: Assets that are not invested in the CPI-linked Bond Fund								
are allocated to Equity Fund with mean return μ , and volatility $\sigma = 20\%$								
The expected return from CPI-linked Fund is $r = 5\%$, with volatility $\eta = 8\%$.								
Correlation between CPI-linked and Equity Fund return is $\rho_{ry} = 0\%$.								
Correlation between personal inflation and Equity Fund return is $\rho_{\pi y} = 25\%$.								

Note that the third correlation figure, $\rho_{\pi y} < 91.66\%$ is forced by the two other values $\rho_{ry} = 0$ and $\rho_{\pi r} = 0.40$. For consistency, we have chosen the same value as before, $\rho_{\pi y} = 25\%$.

Table #3: OPTIMAL ALLOCATION TO CPI-LINKED FUND									
Assuming $W_0 = 33.3$, which is an initial spending rate of $1/33.3 = 3\%$									
Expected	$\rho_{\pi r} = 0.25$			$\rho_{\pi r} = 0.50$			$\rho_{\pi r} = 0.95$		
Investment	Volatility σ			Volatility σ			Volatility σ		
Fund Return	10%	15%	20%	10%	15%	20%	10%	15%	20%
$\mu = 6\%$	0.485	0.686	0.795	0.521	0.707	0.808	0.590	0.746	0.832
$\mu = 8\%$	0.365	0.577	0.710	0.410	0.604	0.728	0.503	0.660	0.763
$\mu = 10\%$	0.325	0.520	0.654	0.377	0.552	0.676	0.486	0.621	0.721
We expect the cost of living to increase by $\pi = 4\%$ per year, with a volatility of $\xi = 3\%$.									
The expected return from the CPI-linked fund is $r = 5\%$ per annum with a volatility of $\eta = 8\%$									
The correlation between the equity fund and the CPI-linked fund is $\rho_{ry} = 0\%$									
The correlation between the cost of living (liability) and the investment fund is $\rho_{\pi y} = 25\%$									

This table examines the low (3%) spending rate case.

Inflation Rate: Wage-earners (W) vs. Elderly (E)

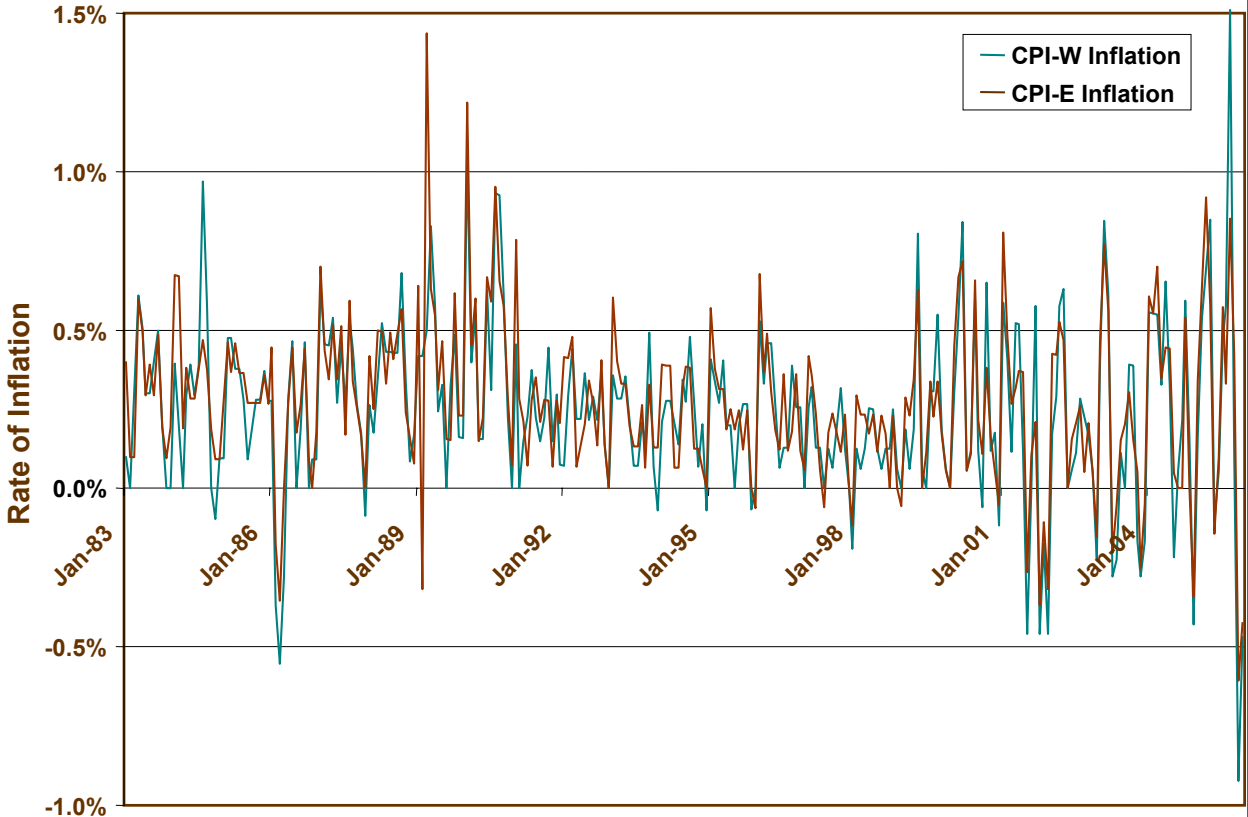


Figure #1

Does Inflation for Wage-earners (W) and Elderly (E) Move Together? Monthly Inflation

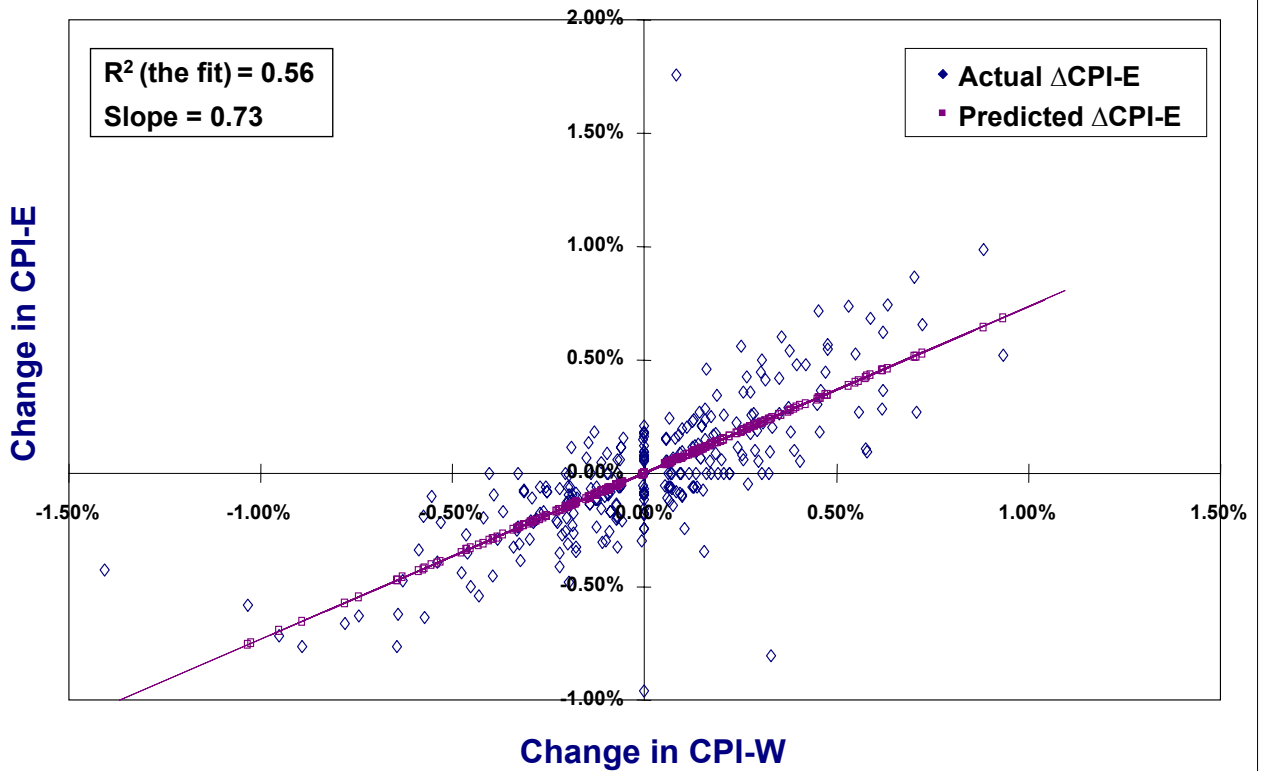


Figure #2

**Value of \$1 Invested in Inflation Linked Bond Fund*
vs.
the Nominal Value of a \$1 Liability adjusted for CPI-E**

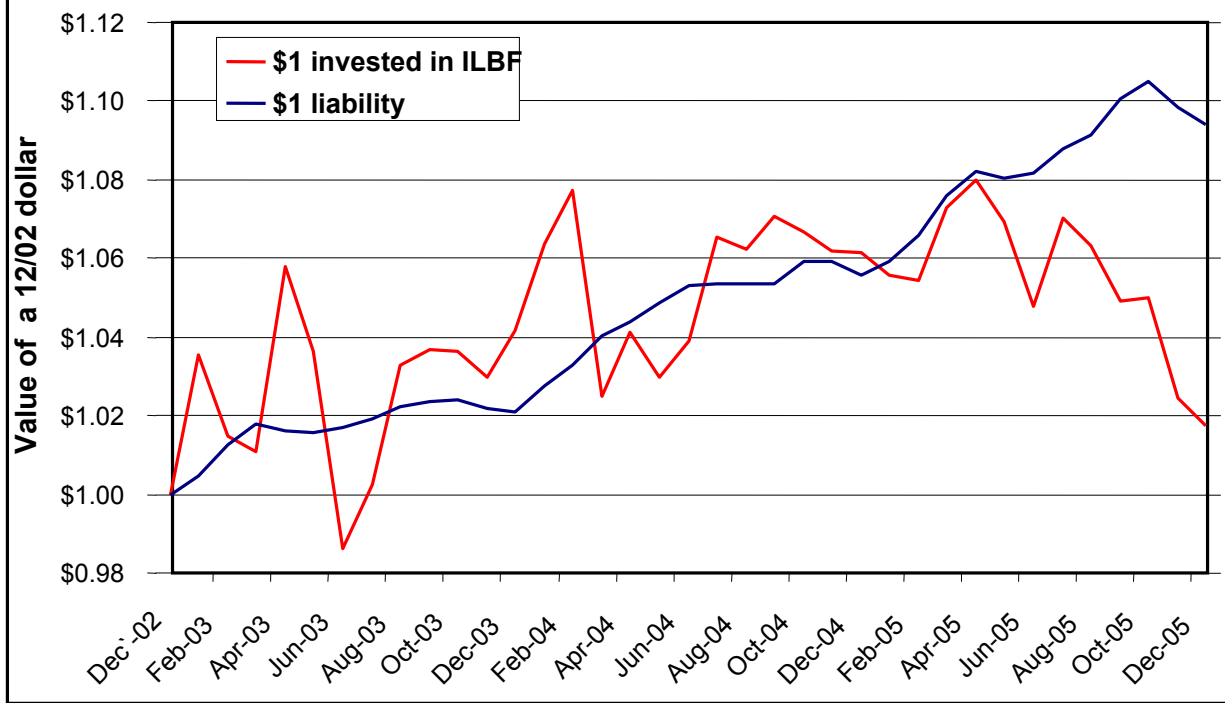


Figure #3

*Based on the averaged returns of four ILBFs

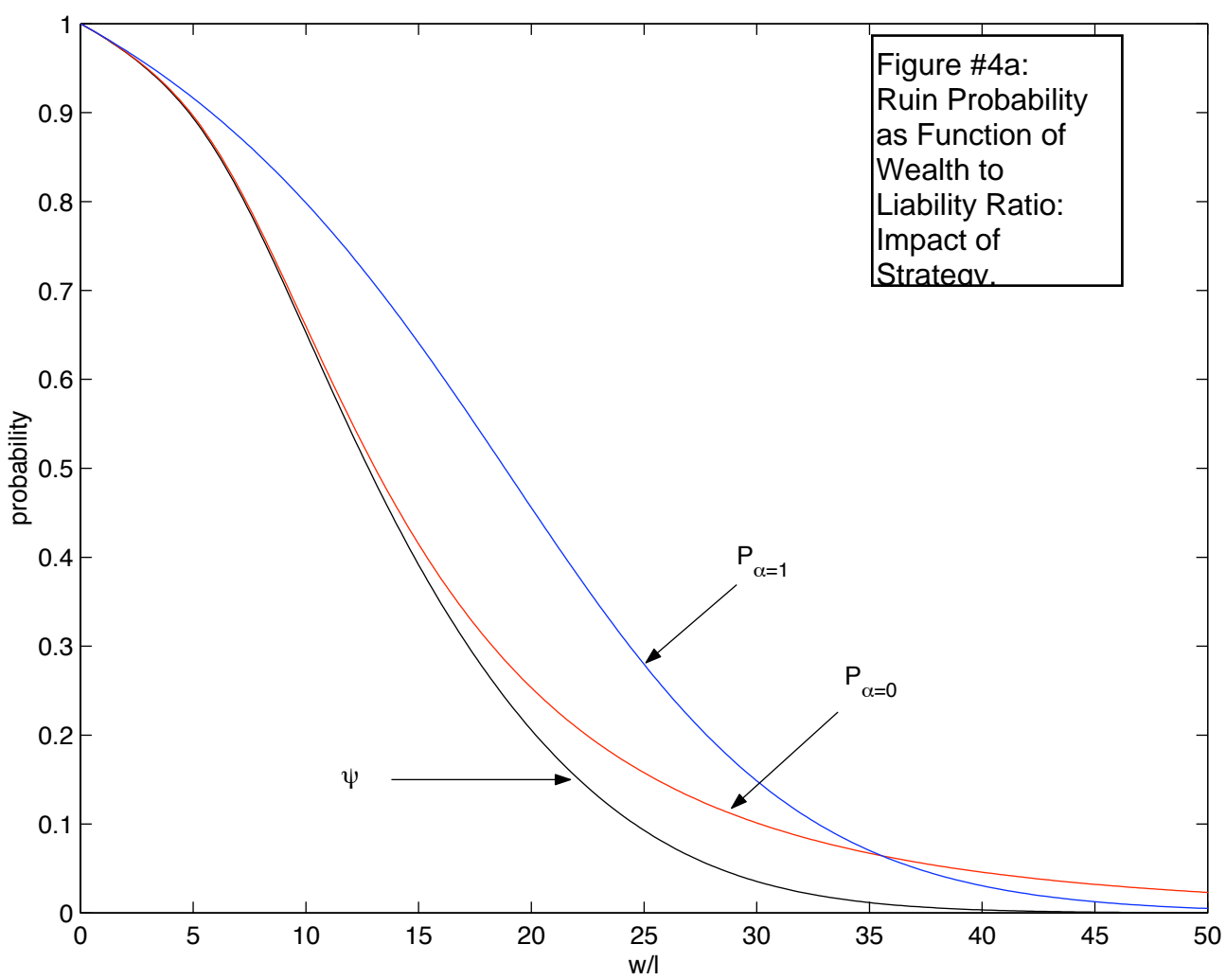


Figure 4b:
Ruin Probability
as Function of
Wealth to
Liability Ratio:
Impact of Age

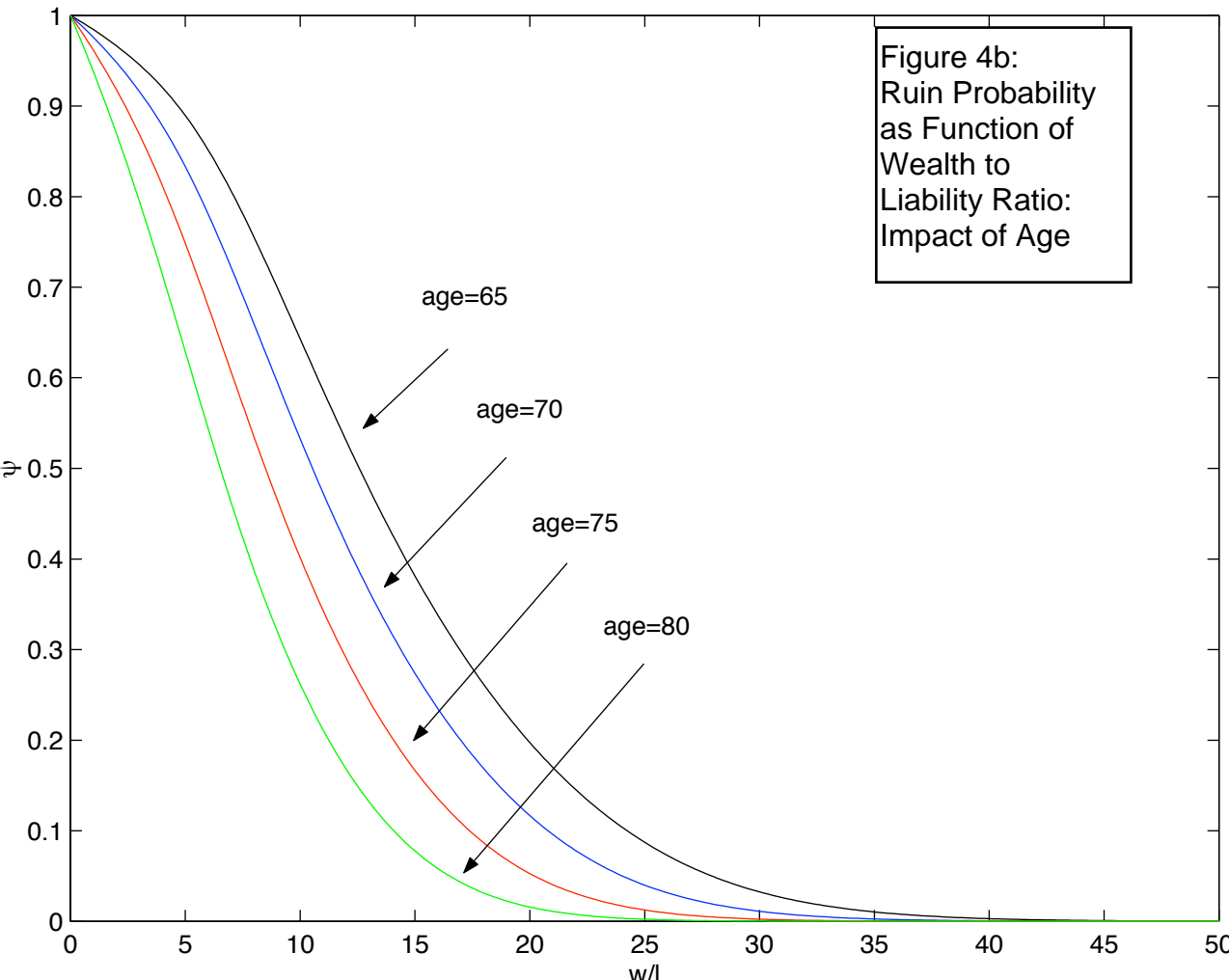
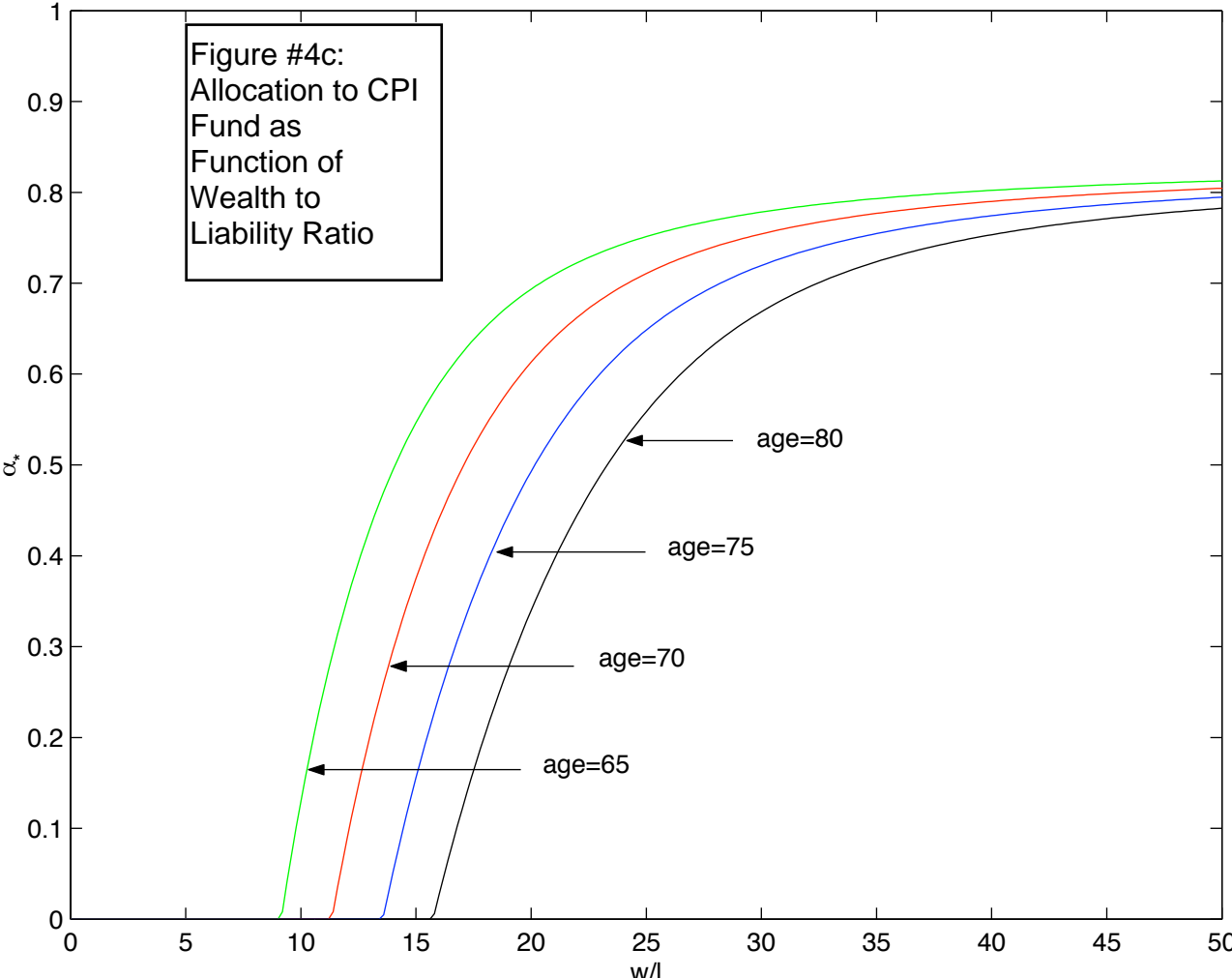
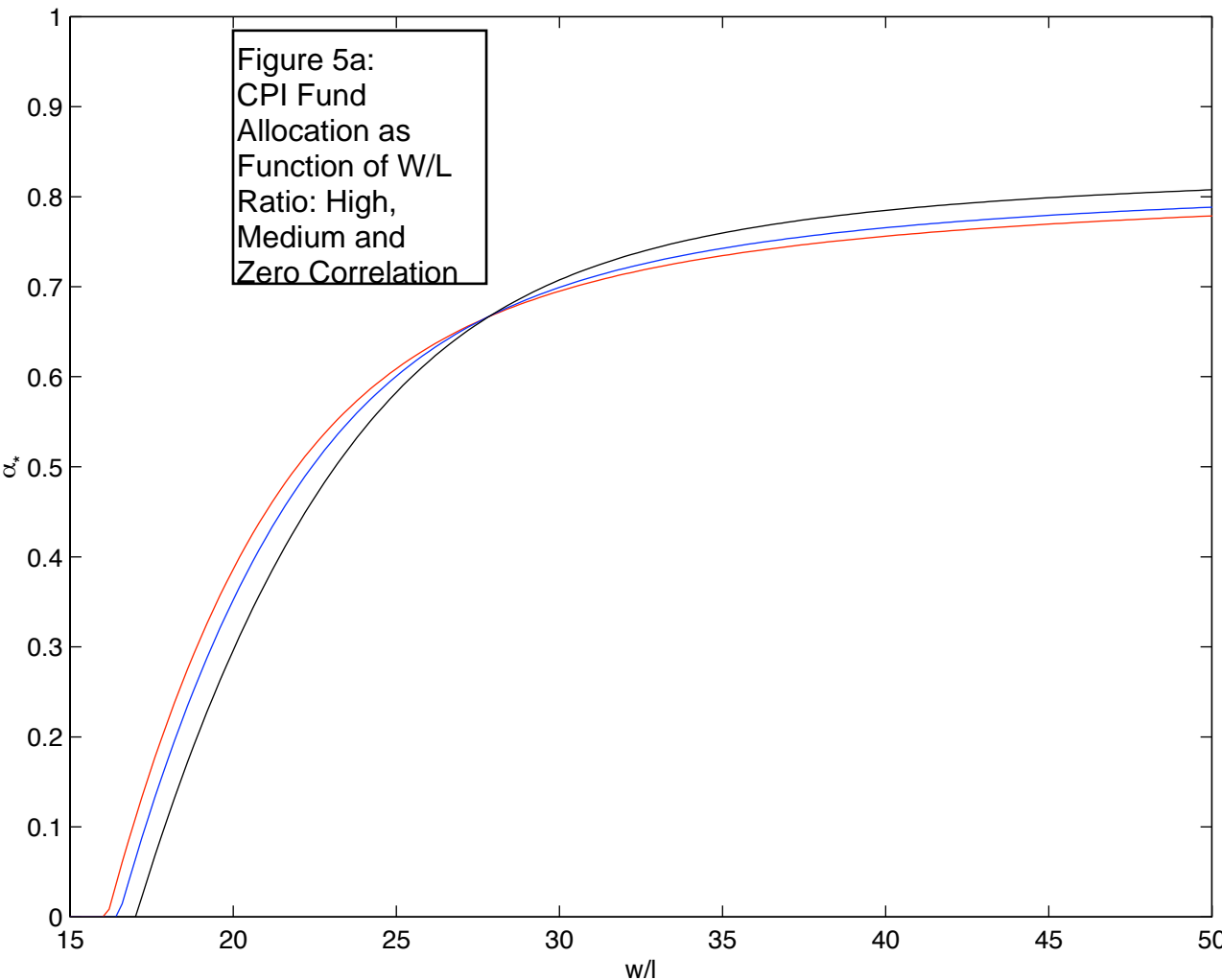


Figure #4c:
Allocation to CPI
Fund as
Function of
Wealth to
Liability Ratio



$\rho_{\pi r} = 0.96$ (black); 0.4 (blue); 0 (red)

Figure 5a:
CPI Fund
Allocation as
Function of W/L
Ratio: High,
Medium and
Zero Correlation



$\rho_{\pi r} = 0.96$ (black); 0.4 (blue); 0 (red)

