

Asset Allocation within Variable Annuities: The Impact of Guarantees

By:

Moshe A. Milevsky¹ and Vladyslav Kyrychenko

ABSTRACT:

The latest generation of variable annuity (VA) contracts contain equity put options plus longevity insurance. Much of the marketing material for these products, claim that these new riders should induce investors to take-on more financial risk. In this paper we examine whether this is indeed the case. Using a unique database we document that policyholders are in fact adopting higher equity exposures when these riders are selected. We also examine the theoretical merits of this advice by deriving the optimal asset allocation in the presence of these guarantees. Our main conclusion is that more aggressive equity allocations can indeed be justified in many, although not all product structures.

¹ Moshe Milevsky is an associate professor of finance at the Schulich School of Business at York University and is the Executive Director of The IFID Centre in Toronto, Canada. He can be reached at milevsky@yorku.ca, Tel: 416-736-2100 x 66014. Vladyslav Kyrychenko is a doctoral candidate at the Schulich School of Business. Both authors would like to acknowledge funding from the IFID Centre and are especially grateful to LIMRA for providing access to their data. The authors would also like to acknowledge helpful comments and discussions with Shlomo BenArtzi, Matthew Drinkwater, Huaxiong Huang, Thomas Salisbury and Eric Sondergeld.

Motivation and Agenda

Variable Annuities (VAs) are close cousins of mutual funds -- which bundle individual securities, such as stocks and bonds, into diversified units or trusts -- but they are formally classified as an insurance policy (since they are sold by insurance companies and contain insurance guarantees) in addition to being registered as a security. In the United States they have provided tax sheltered growth and deferral and they now embed a number of put-like derivatives that provide guarantees on the account value.

As of early 2007 there is more than \$1.2 trillion USD invested in Variable Annuities in the U.S., with gross annual sales in the hundred billion dollar range. This is not a trivial market. Like all insurance riders, and in contrast to standard exchange traded options, insurance companies charge for this downside protection by deducting an ongoing fraction of assets as opposed to an up-front fee. These unique features differentiate the pricing of this derivative security from the standard Black-Scholes approach where the option premiums are paid up-front and in advance. This will be important later when we examine optimal allocations.

The latest generation of variable annuity contracts have been financially engineered to provide an assortment of lifetime income guarantees that are meant to protect the policyholder against -- what the industry has coined -- the “sequence of returns” risk. This refers to the chance that a retirement portfolio from which cash is being withdrawn, suffers early losses. The common denominator of all these insurance riders is that they contain an implicit put option on financial markets plus some form of longevity insurance, akin to a pure life annuity. Of course, using the concept of put-call parity, they can also be viewed as call options to annuitize at a variable strike price. It is estimated that approximately 70% to 80% of VAs being sold now contain these living benefit riders, which place this in the \$100 billion category.

Much of the promotional material for these products claim that these guarantees should induce investors to take-on more financial risk than they normally would – if they didn’t have these guarantees. In fact, it is common to hear the VA+GMIB referred to as a possible “bond substitute” within a diversified portfolio, or even as a risk-free instrument.

This is precisely the issue at the core of this research paper. Using a unique database of policyholder behavior supplied by the Life Insurance Marketing Research Association (LIMRA), we document that VA policyholders are indeed adopting more aggressive allocations (i.e. higher equity exposures) when these riders are actually selected. We also examine the theoretical merits of this advice by deriving the optimal asset allocation – under a stylized model of these products – in the presence of these optional riders.

We focus specifically on a type of rider called a Guaranteed Minimum Income Benefit (GMIB). While the exact mechanics of this option are explained analytically in the section entitled “Model of Portfolio Choice”, the essence of a GMIB is a market put option that allows the holder of the VA to annuitize the account at a guaranteed rate – which then provides a guaranteed level of lifetime income. The GMIB is closely related to the Guaranteed Minimum Withdrawal Benefit (GMWB) – which is not the focus of this analysis – which is yet another form of put option contained and selected within many variable annuities. See the paper by Milevsky and Salisbury (2006) for a separate analysis of GMWB-based products. Both GMIB and GMWB fall in the category of Guaranteed Living Benefits (GLB). A GLB rider is essentially an insurance rider which provide some sort of portfolio insurance for a VA policyholder, but only once the variable annuity is converted into income.

The GMIB can be selected (or not) when the variable annuity policy is acquired or purchased. This rider gives the holder the ability to annuitize some minimally guaranteed amount at some contractually guaranteed rate. Thus, for example, if a \$10,000 premium is placed into a variable annuity, the insurance company might guarantee that at least \$15,000 worth of life annuity income can be purchased in ten years. The purchase price (or annuity factor) would be specified within the contract, for example \$20 per dollar of lifetime income. So, essentially, this contract would guarantee a life annuity of $\$15,000 / \$20 = \$750$ per year in the worst case scenario. And, if the market value of the (sub accounts within the) variable annuity is worth more than \$15,000 in ten years time, the policy holder can annuitize the market value at market annuity rates.

We obviously don't want to get lost within the minutia of these insurance policies and believe that our research sits squarely within the portfolio choice literature since we are essentially investigating optimal asset allocations, but in the presence of various guarantees. Our main empirical conclusions are that -- as one might expect -- investment guarantees do in fact induce risk-taking behavior (a type of moral hazard), even at advanced ages. But, we also echo a warning note that in some (although not all) cases, this extra amount of risky asset exposure might not be completely justified. In the extreme, this could be akin to buying traditional homeowners insurance that only protects against wind-related damage, but not being aware that flood damage is not covered. In the context of many of these variable annuity riders, the investor truly has to annuitize in order to capitalize on the value of the guarantee.

The remainder of this paper is organized as follows. In the next section we review some of the relevant academic literature on the topic of portfolio choice over the human lifecycle. We then describe our data and provide summary results. The subsequent section provides a very basic analytic model of portfolio choice in the presence of these guarantees and the final section concludes with some additional observations.

Review of the relevant literature.

A number of recent papers have extended the set of decisions included in the portfolio choice problem to highlight the interaction and the risks faced by the household, broadly defined. Thus, for example, Goetzmann (1993), Yao and Zhang (2005) as well as Cocco (2005) focus on the role of the housing portfolio; Campbell and Cocco (2003) focus on optimal mortgage choices; Cairns, Blake and Dowd (2005) examine portfolio choice in defined contribution pension plans, Sundaresan and Zapatero (1997) examine the role of defined benefit pensions, while Dybvig and Liu (2004), Bodie, Detemple, Ortuba and Walter (2004) model the impact of flexible retirement dates; Jagannathan and Kocherlakota (1996) and Viceira (2001) stress the impact of aging; Faig and Shum (2002) are motivated by the demand for illiquid assets; Koo (1998) as well as Davis and Willen (2000) model the role of labor income; Dammon, Spatt and Zhang (2001) focus their attention on capital gains and income taxes; Heaton and Lucas (2000a) focus on the role of background risk. Others go back to basics and extend portfolio choice models to include more

sophisticated (and realistic) processes for investment returns, such as Chacko and Viceira (2005) or time-varying and mean-reverting risk premiums, such as Kim and Omberg (1996) or Detemple, Garcia, Rindisbacher (2003).

The common denominator in the above papers – and a frequently debated question -- is how portfolio allocations to risky assets (stocks) should evolve with investors' age. Finance practitioners consistently recommend lowering stocks content of one's portfolio with increasing age. Much of this is at the heart of recent lifecycle funds promoted by mutual fund companies. A popular rule of thumb is to have the share of equities equal to 100 minus investor's age. Academic financial economists turned to this issue mostly in the last decade. Several models of portfolio choice that include human capital, e.g. Bodie, Merton, and Samuelson (1992), Jagannathan and Kocherlakota (1996), Cocco, Gomes, and Maenhout (2005), Gomes and Michaelides (2005) by and large support the recommendation to decrease equity share in the portfolio with increasing age, although the generated profiles are not necessarily monotonic. Recall the classical result by Merton (1969, 1973) that age should not matter in the absence of human capital considerations.

A number of these papers have established some stylized empirical facts. Several studies investigate the determinants of the stocks share in investors' portfolios, in particular the age effect. Relevant to this paper, most of these studies document a declining age-equity share profile, for example Bodie and Crane (1997), VanDerhei, Galer, Quick, and Rea (1999), Agnew, Balduzzi, and Sunden (2003), and Curcuru, Heaton, Lucas, and Moore (2004). Others document a hump-shaped pattern, with a declining part starting at the ages of 50-60. Examples of these would include Yoo (1994), Bertaut and Starr-McCluer (2001), and Faig and Shum (2006).

It is important to remind the reader that Ameriks and Zeldes (2004) show that the age to equity share profile is sensitive to the model specification. In fact, any regression explaining portfolio choice with age can include only two of the possible three variables: age, time, and cohort. While Ameriks and Zeldes (2004) find a hump-shaped relationship in a regression with age and time effects, they report an increasing allocation to stocks/equity with age in a regression with age and

cohort effects. The overwhelming majority of researchers, however, consider a cohort effect the least significant among the three and make the assumption that it is equal to zero.

And, just to preview some of our main results, we also find a declining (equity, stock) risk allocation within the account as policyholders age, but the age effect is substantially weakened once the insurance rider -- which provides the GLB, defined above -- is selected. Our estimation results are displayed in Table #3 and Table #4, and will be explained later.

Our Unique Data

As mentioned above, the data that we employ in our empirical analysis was provided by LIMRA. They collected contract and product information from 10 member companies that sold variable annuity policies within the period between January 2000 and June 2004; the policies had to be in force as of June 2004 and had to offer at least one GLB rider at the time of purchase. As a result, LIMRA's dataset contains information on 812,367 individual variable annuity contracts.

LIMRA's dataset has several advantages compared to most other sources of financial micro level data. The two most popular data sources employed in household portfolio choice literature are Survey of Consumer Finances (SCF) and Panel Study of Income Dynamics (PSID). First, LIMRA's dataset has a definite size advantage. It provides data on more than 812,000 annuity contracts. In comparison, PSID contains panel data on about 8,000 families while SCF has information on 4,000+ households in each of its triennial surveys. Second, LIMRA's data come directly from insurance company's original contracts and therefore are much more accurate than self-reported information from the surveys. Third, LIMRA's data provide much higher level of detail with respect to asset allocation choices (e.g., amounts invested in small-cap, medium-cap, large-cap, or international stock funds, investment grade or high-yield bond funds, balanced funds, etc.) Finally and most importantly, LIMRA's dataset is unique in terms of providing information for asset allocation within variable annuities with and without GLB riders.

However, as was rightfully argued in Campbell (2006) there is no perfect source of household financial data. We have to mention some caveats of LIMRA's dataset, such as insufficient

information about investors' personal characteristics and lack of data on their other investment accounts.

LIMRA's dataset contains 60 variables providing information about investors and contract characteristics, such as: investor's age, gender, state of residence, distribution intermediary channel; account value and its breakdown into amounts invested in several types of funds; whether one of the three GLB RIDER types was selected as well as the features of each type of GLB riders.

We had to eliminate records with missing age information and those records for which the values of different sub-accounts do not add up to the account value; this reduced our sample from 812,367 to 679,579 observations. We further limit our sample to those investors who (i) either selected no GLB rider or (ii) selected the Guaranteed Minimum Income Benefit (GMIB) only. We do this for two reasons. First, compared with other GLB riders, such as Guaranteed Minimum Withdrawal Benefit (GMWB) or Guaranteed Minimum Accumulation Benefit (GMAB), GMIB is the closest equivalent to a life annuity with longevity insurance. Second, it is by far the most popular GLB rider selected by individuals. Of all the contacts with at least one GLB rider selected, the contracts where only GMIB was selected represent 95 percent. As the result, we have a “clean” dataset of 660,336 observations with either GMIB or no GLB riders selected.

Analysis of Results.

We now present the relationship between asset allocation and age for investors who selected GMIB vs. those who did not select any guaranteed living benefit (GLB). More specifically, we consider the percentage of VA account invested in high and medium risk (HMR) assets (a.k.a risky assets) as a function of investor's age. According to our definition, we regard the following funds as high or medium risk: large-cap, mid-cap, and small-cap stock funds, high yield bond fund, balanced fund, specialty/sector fund, international equity fund. On the other hand, we view investments in investment-grade bond fund, money-market fund, and fixed fund, as low risk (LR).

We now focus our analysis on the two companies (anonymously referred to as company A and company B) with the highest number of annuity policies in our sample, 170,462 and 126,118 annuity contracts respectively. These two companies combined represent 52 percent of VAs with selected GMIBs. Figure #1 and Table #1 present the percentages invested in risky assets – for the two companies we analyzed - for the following age groups in five year buckets. In both cases, two age vs. HMR lines are presented: one for investors who selected GMIB and another for those who did not select any GLB rider.

[Figure #1 and Table #1 placed here]

As can be observed from company A results, for investors who did not select any GLB rider, the percentage invested in HMR assets declines with age almost monotonically from 77 percent in the under-age 40 group to 54 percent in the above-age 80 group. This is very different from the age-HMR profile for those investors who selected GMIB. Among the latter, the percentage invested in HMR assets declines with age much slower until the age 71-75, and then starts to rise. Overall, their exposure to HMR assets is much higher, being in the range between 80 and 86 percent. We conclude that additional HMR (risk) exposure in the group of GMIB selectors in company A is between 11 and 31 percent.

The impact of age on the HMR allocation graph of company B displays a similar trend. The allocation to risky assets for investors who selected a GMIB is substantially higher and declines with age much slower than for investors who did not select any GLB rider. Interestingly, the share of assets invested in HMR assets, is significantly lower in company B than in company A for any age group. Furthermore, in company B, for investors who selected GMIB, this share is very stable regardless of age. These differences should be attributed to the different investment choices available in each company, as well as to different asset allocation restrictions applied when a GMIB rider is selected. Additional HMR exposure in the group of GMIB selectors in company B is between 5 and 18 percent.

[Figure #2 and Figure #3 placed here]

Finally, Figure #2 and Figure #3 illustrate the impact of the distribution channel on the selection of HMR versus LR assets. What we mean by distribution channel is the type of intermediary that sold the variable annuity to the customer in question. We find stark differences in the HMR equity exposure -- both with and without the selection of the GMIB -- depending on whether the VA was purchased through a bank, independent financial advisor, wirehouse broker, etc. For example, in company A, the policies with highest allocation to risky assets -- with or without GMIB - were sold by financial planner. On the other hand, policies sold through a bank have the biggest difference between the percentages allocated to HMR for investors who selected GMIB vs. those who did not. In company B, VA's with highest HMR allocations (where no GLB RIDER were selected) were sold by stock broker and the policies with GMIB selected -- by financial planner. Here the largest gap in allocations to risky assets was in the policies sold by stock broker. Of course, this is not to say that the any of the channels necessary influence or cause the allocation, this could obviously be a clientele effect. People who are more "conservative" and hence likely to hold less HMR assets might be more likely to purchase the VA through a bank, and vice versa. In other words, the most likely explanation for the asset allocation distribution channel effect is likely to be the type of customer who uses these intermediaries as opposed to the intermediaries themselves.

Multivariate Statistical Analysis

In the next step we check whether the relationship between age and percentage invested in HMR assets observed in Table #1 remains unchanged in the presence of other variables that could explain HMR percentage. To do this, we employ a linear model with percentage invested in HMR assets as the dependent variable. In this model, we use the following explanatory variables besides investor's age.

1. Age squared (AGE2). Captures non-linearity on the age to HMR relationship. Previous studies in the literature have frequently used this specification.
2. Log of account value (LOGACC). Can be viewed as a proxy for investor's wealth, although we are fully cognizant of the fact that we do not know the investors other

holdings, which could be substantial. Other studies often find that wealthier investors have more aggressive asset allocations, which is why we include as well.

3. Gender (dummy variable MALE). A number of previous studies document and argue that males are generally considered to be more overconfident compared to females, which is why they may have a tendency to invest proportionally more in risky assets. Whether or not this is the case/reason, we included this dummy variable to test the presence of a gender-based effect.
4. Inside or Outside an IRA. We use a dummy variable to capture the tax status of the underlying account. The variable is equal 1 if the variable annuity is classified as an IRA, a.k.a personal pension plan and zero otherwise. Since investments inside IRAs are tax-sheltered, we may expect to see a higher share of fixed income assets in those accounts; the interest income of which is taxed at a higher rate than dividends or capital gains on stocks. Remember, that variable annuities are tax sheltered investment already, so theoretically it should not make a difference in terms of the tax implications.
5. Distribution channel. Intermediary by which the variable annuity contract was purchased. We employ four dummy variables, CAREER, STBR, FINPL and BANK, which represent the following distribution channels: insurance agent, stockbroker, financial planner, and bank employee, correspondingly. The omitted category is independent agent. This variable controls for the possible impact and influence of the intermediary on the percentage of risky assets selected
6. Company. As the share invested in risky assets can be company-specific, we control for this by using seven dummy variables for each of the seven companies (out of original ten) that sold VA policies with GMIB riders. The omitted category is the three companies with the lowest number of VA contracts with GMIB.

We run linear regression on two sub-samples. One sub-sample contains the data on variable annuity contracts for which no GLB rider was selected (Model 1), while the second regression only includes the contracts where variable annuity holders selected a particular type of GLB RIDER, namely the Guaranteed Minimum Income Benefit GMIB (Model 2). The results of both models are presented in Table #2.

[Table #2, placed here]

As judged by the negative coefficient on AGE, the share invested in risky assets declines with age in both sub-samples. However, it declines much faster for those investors who did not select any GLB RIDER, as the AGE coefficient in Model 1 is three times higher in absolute value than in Model 2 and their difference is highly significant based on t-test. Moreover, AGE2 coefficient is negative in Model 1 but positive in Model 2. This suggests that the share invested in risky assets is a concave function of age that declines faster for older ages for investors who did not select any GLB RIDER. For those who selected GMIB, however, decline with age is much slower and there is even a reversal for older ages. These results are generally in line with Figure # 1.

To assess whether regression results overall are different for GMIB selectors vs. non-selectors, we combine the two sub-samples into one pooled sample and perform what is often called a Chow F-statistic test, which lends support to the argument that the differences are statistically significant.

Besides age, several other variables prove to be significant in explaining the percentage of an account invested in HMR assets in both models. As expected from previous literature referenced above, men invest more aggressively than (i.e. with higher concentrations of equity compared to) women. Variable annuities that are part of a tax-sheltered (IRA) personal pension plan have a lower share of risky assets and a higher proportion of low-risk bonds. This would be an intuitively pleasing result – i.e. bonds are more likely to be placed in tax shelters – where it not for the (perhaps not) widely known fact that variable annuities are already tax sheltered. Another odd result, however, is that the account size (wealth level) is negatively associated with an allocation to HMR (risky) assets.

The coefficients on the distribution channels dummy variables lead one to the conclusion that for investors who did not select a GLB rider, financial planners induce (recommend?) higher allocation to risky assets than other distribution channels. For those who selected the GMIB, however, relatively riskier asset allocation is induced (recommended?) by stock brokers. As for

the company dummies, all of them are statistically significant. However, since the companies are anonymous in our study, we will not delve in to the GMIB contract details (some more lucrative than others) which might drive this particular result. Instead, we repeat our regression analysis for two companies with the largest number of variable annuity policies in our sample. The results of the two separate regressions are presented in Table #3.

[Table #3, placed here]

In terms of AGE to HMR (risky equity) relationship, these results are similar to those presented in Table #2. The share of risky assets declines with AGE much faster for investors who have not selected a Guaranteed Living Benefit. Again, one can see some increase of HMR share in older ages for investors who selected GMIB. Male investors generally allocate more to risky assets. In terms of account market value, however, the results are different depending on the company. In company A, the market value of the account is positively related to the share of risky assets, whereas in company B, the relationship is negative (similar to the result for the whole sample). Interestingly – and in contrast to the pooled results -- in both companies the share invested in HMR is higher when the variable annuity is within tax-deferred account, but only when the GMIB is not selected. But, when the GMIB is selected the IRA status is associated with “more bonds” and less HMR asset classes.

A Model of Portfolio Choice with a GMIB:

In this section we construct a stylized model of portfolio choice in the presence of a Guaranteed Minimum Income Benefit (GMIB) option. Our objective is to derive a measure for the amount of “extra risk” a rational (utility maximizing) investor would be willing to take when granted a GMIB option that protects his/her downside risk.

We start by postulating a generic investor with W_0 of initial wealth that is currently optimally allocated α^* to risky assets, and $1 - \alpha^*$ to safe assets. These allocations take place within a variable annuity, but without any extra riders. The risky and safe assets correspond to the HMR and LR funds described in the earlier section. By selecting this particular allocation – which we

assume is optimal -- the investor has revealed his explicit risk preferences. And, under the classic Merton (1969, 1970) model that assumes constant relative risk aversion (CRRA) and lognormal asset returns, the implied coefficient of relative risk-aversion γ will be equal to $(\mu - r)/\sigma^2$, where μ is the expected return, σ is the volatility and r is the risk-free rate of return. For example, an investor with a pre-existing allocation of 50% HMR and 50% LR reveals a risk-aversion of exactly 4, when the expected return from the HMR investments is 9%, the volatility is 15% and the risk-free (LR) rate is 4.5%.

We then take this so called $\alpha^* \Rightarrow \gamma$ investor and compute (solve for) how he might change his allocation to the HMR asset class if he were to hypothetically be granted an annuity put option which underlies the GMIB.

More specifically, let α^{**} denote the new (presumably) optimal allocation to the HMR asset class when the GMIB is “wrapped” around the investment account. Recall the α^* is the original (optimal) allocation in the absence of this put option guarantee. The difference between α^{**} and α^* , which we define as ε , is the incremental allocation that is theoretically justifiable, based on the presence of the GMIB.

Our agenda in this section is to investigate the behavior of this ε quantity as a function of the various underlying contractual parameters, such as the strike price of the embedded option, the pre-existing allocation α^* , and other exogenous capital market parameter assumptions.

Recall the GMIB guarantees the ability to convert or annuitize (in the worst case scenario) the guaranteed amount $W_0 e^{\eta T}$ at a pre-specified rate denoted by g_x , where η is a guaranteed investment return and T is the contract horizon. Alternatively, of course, the investor can annuitize the account value \tilde{W}_T at the-then market rate denoted by \bar{a}_x . The quantity \tilde{W}_T is obviously unknown in advance and depends on both the selected allocations of the investor as well as the random performance of underlying market sub-accounts. The subscript x on both annuity factors denotes the age at which the life annuity is priced or issued, for example age 70 or 75. We refer the interested reader to Milevsky (2006) for a detailed explanation of the

actuarial pricing underlying the annuity. Typical market values of \bar{a}_x under the current interest rate environment might be \$10.2 at the age of $x=70$, 8.44 at the age of $x=75$ and 6.72 at the age of $x=80$. This is the cost of \$1 of annual lifetime income, at the various purchase ages.

One can also think of g_x as the strike price of the GMIB option, although it is not really a put option to sell in the conventional sense, but more of an exchange rate between a guaranteed amount and lifetime income. Either way the GMIB option pays-off, or promises lifetime income in the amount:

$$I := \max \left[\frac{W_0 e^{\eta T}}{g_x}, \frac{\tilde{W}_T(\alpha)}{a_x} \right] \quad (\text{eq.1})$$

The justification for (eq.1) is as follows. If the underlying market and sub-accounts perform poorly (i.e. earns less than η per annum during the T -year waiting period), the investor is guaranteed the ability to annuitize $W_0 e^{\eta T}$ at the guaranteed annuity factor rate of g_x . On the other hand, if the market value of the account $\tilde{W}_T(\alpha)$ at time T , is greater than the guaranteed amount $W_0 e^{\eta T}$, the investor can simply annuitize the (higher) account value, at the-then market annuity rates \bar{a}_x . In fact, he does not have to purchase this life annuity from the company that issued the GMIB. He can take his money anywhere and annuitize in the open market. Of course, all of this assumes the investor actually (understands and) wants to annuitize, which is a big un-explored assumption at this stage, which we deftly side-step.

Note once again that $\tilde{W}_T(\alpha)$ is partially under the control of the investor and depends on the asset allocation vector α , which is to be determined. Another way to express this quantity is: $\tilde{W}_T(\alpha^* + \varepsilon)$, where α^* was the original allocation in the absence of the GMIB option. Either way, we have now reached the control and optimal portfolio choice aspect of the problem.

Finally, we multiply the guaranteed lifetime income denoted by I , by the-then market annuity factor \bar{a}_x to convert this flow into a lump-sum value at the horizon time T . The objective is to

locate an asset allocation vector α^{**} , or incremental allocation ε that maximizes expected utility of wealth.

$$U^{**} = \max_{\alpha} E[U(I\bar{a}_x)] \quad (\text{eq.2})$$

The intuition, once again, is as follows. Imagine there is a liquid secondary market for guaranteed lifetime income. In theory the annuitant could de-annuitize the guaranteed income I and obtain a lump-sum in the amount of $I\bar{a}_x$. Thus, the true expected utility of wealth is as displayed in equation (eq.2). Indeed, if the option expires out-of-the-money and the market value of the lifetime income $\tilde{W}_T(\alpha)/\bar{a}_x$ is greater than the guaranteed amount of lifetime income $W_0 e^{\eta T} / g_x$, the mark-to-market value is simply $\tilde{W}_T(\alpha)$ itself. On the other hand, if the option expires in-the-money, the guaranteed lifetime income will kick-in and provide income in the amount of $W_0 e^{\eta T} / g_x$ and the market value of this will income stream will be: $\bar{a}_x W_0 e^{\eta T} / g_x$. The objective then is to locate an asset allocation that maximize the expected utility of this uniquely defined wealth.

Under the (new) optimal allocation, the expected return from the investment account will be $\alpha^{**} \mu + (1 - \alpha^{**})r - f$, where μ denotes the expected return from the HMR funds, r denotes the LR rate and the new symbol f denotes the extra fee (a.k.a. mortality and expense charge) for the GMIB.

We located the optimal allocation and the incremental justifiable risk allocations by simulation, since an analytic approach is impossible and the numerical implementation is equally cumbersome. More specifically, we focused our computational attention on simulations for which the GMIB annuity factors g_x are within the vicinity of market annuity factors \bar{a}_x , which can be viewed as a fair GMIB case. We also generated a few in which the GMIB annuity factors were set-back relative to market based annuity rates, so that: $g_x < \bar{a}_x$. Also, for the majority of our simulations we assumed that the guaranteed return η embedded within the GMIB was 6% per annum. Finally, we started with a 55 year-old investor who purchases a GMIB with a 15 year

horizon. At the age of 70 he plans with 100% certainty to annuitize the account, either under the guarantee (if the HMR asset earns less than 6% over the next 15 years) or under the market rates, generating lifetime income of I .

Our results for the allocation α^{**} that maximizes CRRA utility, relative to the original allocations α^* are displayed in the following table.

[Table #4, Placed Here]

A few things are worth noting from the table and our extensive simulations. The “positivity” of the ε variable – in other words, the justification for additional risk exposure – is dependent on the strike price of the option. The strike price within a GMIB is not immediately obvious and unrelated to the contract’s guaranteed investment return, which we labeled by η , and was in the vicinity of 6% for most of our examples. (Note that sometimes this is expressed with simple compounding, in which case the value must be converted to the true annualized return.)

Our point here is more than just “the devil is in the details.” Rather, there can be remarkable heterogeneity between various GMIB contract provisions that all seem to offer a 6% guaranteed return.

In the course of our extensive simulations – to locate the optimal allocation -- we observed a number of interesting results. First, in many cases the optimal allocations are labeled “corner solutions”. Basically, if the GMIB annuity factor is favorable to the policyholder, he takes on as much risk as possibly allowed by the contract. In a sense the policyholder has been granted a put option and he maximizes the personal value of this option by taking-on as much risk as possible. This is consistent with the empirical evidence, but assumes a 100% annuitization rate.

On the other hand, if the contractual provisions are less favorable, which in our cases is denoted by a higher parameter value of g_x , the optimal allocations in the presence of the GMIB are no longer corner solutions. In some cases the additional risk exposure levels are only on the order of 5% to 10%.

We conclude this section by cautioning the reader that our very simple model of optimal asset allocation within a GMIB structure only scratches the surface of more accurate representations of the dynamic control problem. To name just a few improvements, one would have to incorporate the American-style optionality of when to annuitize, as well as the stochasticity of interest rates and perhaps even the credit risk of the insurer, in the event the market collapses.

While we do not offer a complete theory of GMIB option pricing and utilization, we believe that our result is robust: more risk is acceptable provided the strike price is sufficiently near the money.

Conclusion and Final Thoughts:

Portfolio choice and optimal asset allocation over the lifecycle is a topic that continues to attract both academic and practical interest. Indeed, there are a number of normative and prescriptive models on how individuals *should* invest as a function of their risk aversion and as they age. Likewise, there is empirical and anecdotal evidence that younger investors tend to hold more aggressive equity (risky) allocations, which tends to decline over time as people approach retirement. In this paper we look at things from a slightly different perspective. We examine how actual allocations to risky assets change when individual are given “downside protection” in the form of options to annuitize.

More specifically, we examine asset allocation inside variable annuity (VA) products in which certain insurance riders are available that give investors the option to annuitize at some favorable rate. We have over half a million policyholder accounts spanning the ages from 40 to 80 in which we can observe (a) their asset allocation, and (b) whether or not certain riders were selected.

Our main empirical conclusion is that individuals will invest more aggressively when they are granted this type of put option. Indeed, to anyone trained in the valuation and pricing of American-style derivative securities, this idea is axiomatic. However, what is less obvious is that

these put options are not money-back guarantees but rather contingent on annuitization. In other words, the only way to exercise this put is to irreversibly annuitize the contract (at the strike price) in exchange for lifetime income. Thus, if there is some exogenous propensity to avoid annuitization, despite its welfare enhancing properties, it remains to be seen whether these put holders will in-fact exercise their options if-and-when they expire in the money.

References:

Agnew, J., P. Balduzzi, and A. Sunden (2004), Portfolio choice and trading in a large 401(k) plan, *American Economic Review*, 93(1), 193-215.

Ameriks, J., and Zeldes, S. P. (2004), How do household portfolio shares vary with age? *Working paper*, Columbia University.

Bertaut, C., and M. Starr-McCluer (2001), *Household portfolios in the United States*, in *Household Portfolios*, ed. by L. Guiso, M. Haliassos, and T. Jappelli. MIT Press, Cambridge.

Bodie, Z., and D.B. Crane (1997), Personal investing: advice, theory, and evidence, *Financial Analysts Journal*, 53(6), 13-23.

Bodie, Z., R.C. Merton, and W. Samuelson (1992), Labor supply flexibility and portfolio choice in a life cycle model, *Journal of Economic Dynamics and Control*, 16, 427-49.

Bodie, Z., J. B. Detemple, S. Otruba, and S. Walter (2004) Optimal consumption--portfolio choices and retirement planning, *Journal of Economic Dynamics and Control*, 28 (6): 1013-1226.

Campbell, J.Y., and J.F. Cocco (2003), Household risk management and optimal mortgage choice, *Quarterly Journal of Economics*, 118, 1149-1194.

Campbell, J. and Viceira, L. (2002), *Strategic asset allocation: Portfolio choice for long term investors*, Oxford University Press.

Cocco, J.F. (2005), Portfolio choice in the presence of housing, *Review of Financial Studies*, Summer 2005, 18, 535-567.

Cocco, J.F., F.J. Gomes, and P.J. Maenhout (2005), Consumption and portfolio choice over the life cycle, *Review of Financial Studies*, Summer 2005, 18, 491-533.

Curcuro, S., Heaton, J., Lucas, D., and Moore, D. (2006), Heterogeneity and portfolio choice: Theory and evidence, (forthcoming), in Y. Ait-Sahalia and L.P.Hansen, eds. *Handbook of Financial Econometrics*, Elsevier Science, Amsterdam.

Dammon, R. M., C.S. Spatt and H.H. Zhang (2001), Optimal consumption and investment with capital gains taxes, *Review of Financial Studies*, Vol. 14: 583-616

Faig, M., and P. Shum (2002), Portfolio choice in the presence of personal illiquid projects, *Journal of Finance*, 57, 303-328.

Faig, M., and P. Shum (2006), What explains household stock holdings, *Journal of Banking and Finance*, 30(9), 2579-2597.

Goetzmann, W.N. (1993), The single family home in the investment portfolio, *Journal of Real Estate Finance and Economics*, 6, 201-222.

Gomes, F., and A. Michaelides (2005), Optimal life-cycle asset allocation: Understanding the empirical evidence, *Journal of Finance*, 55 (2), 869-904.

Hakansson, N. H. (1969), Optimal investment and consumption strategies under risk, an uncertain lifetime, and insurance, *International Economic Review*, Vol. 10, No.3, 443-466.

Heaton, J., and D.J. Lucas (2000), Portfolio choice in the presence of background risk, *The Economic Journal*, 110 (January), 1-26.

Heaton, J., and D.J. Lucas (2000), Portfolio choice and asset prices: The importance of entrepreneurial risk, *Journal of Finance*, 55, 1163-1198.

Jagannathan, R., and N.R. Kocherlakota (1996), Why should older people invest less in stocks than younger people?, *Federal Reserve Bank of Minneapolis Quarterly Review*, 20, 11-23.

Kim, T.S. and E. Omberg (1996), Dynamic non-myopic portfolio behavior, *Review of Financial Studies*, Vol. 9: 141-161.

Koo, H.K. (1998), Consumption and portfolio selection with labor income: a continuous time approach, *Mathematical Finance*, Vol. 8(1): 49-65.

Merton, R. C. (1969), Lifetime portfolio selection under uncertainty: The continuous time case, *Review of Economic Studies*, Vol. 51: 247-257.

Merton, R. C. (1971), Optimum consumption and portfolio rules in a continuous time model, *Journal of Economic Theory*, Vol. 3 (December): 373-413.

Milevsky, M.A. and T.S. Salisbury (2006), Financial valuation of guaranteed minimum withdrawal benefits (GMWBs), *Insurance: Mathematics and Economics*, Vol. 38, pg. 21-38.

Milevsky, M.A. (2006), *The Calculus of Retirement Income: Financial Models for Pension Annuities and Life Insurance*, Cambridge University Press, UK

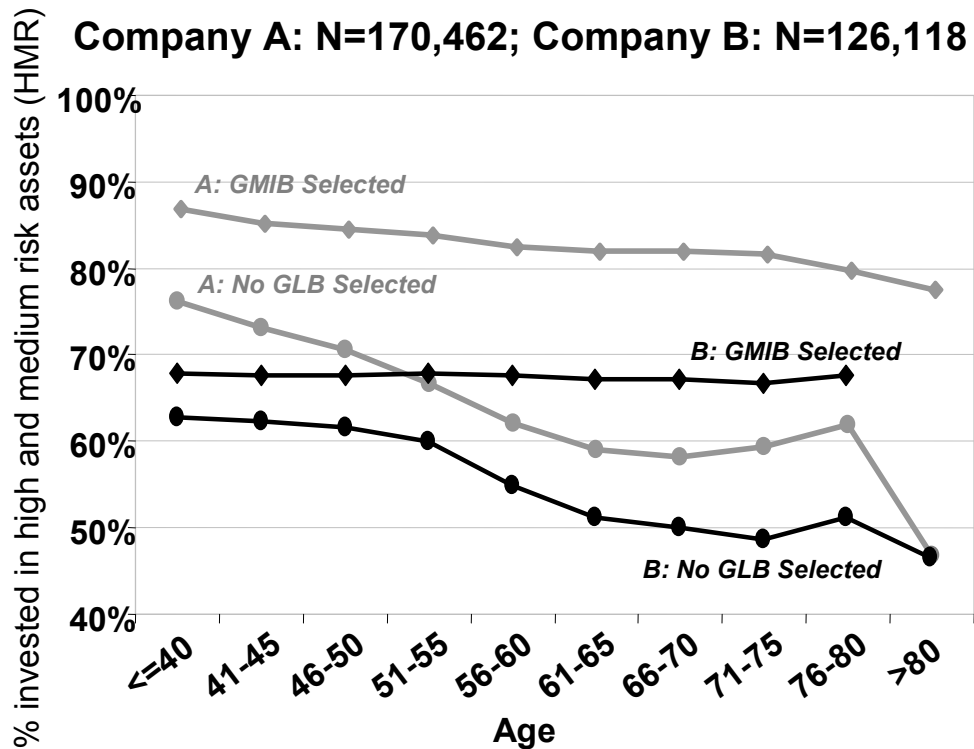
VanDerhei, J., R. Galer, C. Quick, and J. Rea (1999), 401(k) plan asset allocation, account balances, and loan activity, *EBRI Issue Brief*, (205).

Viceira, L. (2001), Optimal portfolio choice for long-horizon investors with nontradable labor income, *Journal of Finance*, 55, 1163-1198.

Yao, R., and H.H. Zhang (2005), Optimal Consumption and Portfolio Choices with Risky Housing and Borrowing Constraints, *Review of Financial Studies*, Spring 2005, 18, 197-239.

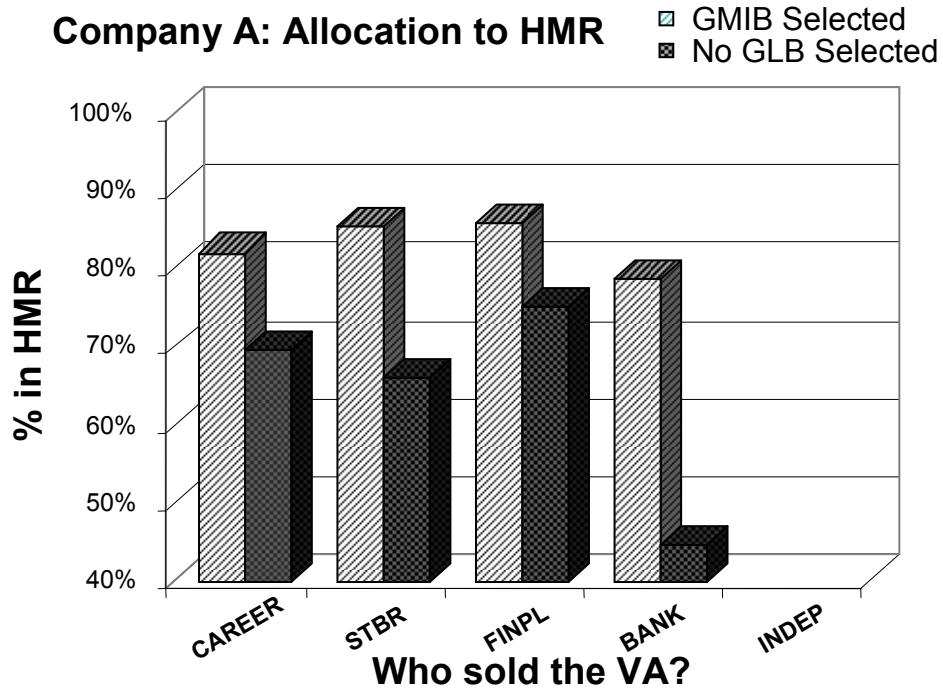
Yoo, P.S. (1994), Age dependent portfolio selection, Federal Reserve Bank of St. Louis *Working Paper No.* 94-003A.

Figure #1: Allocation to Risky assets, with and without GMIB option.



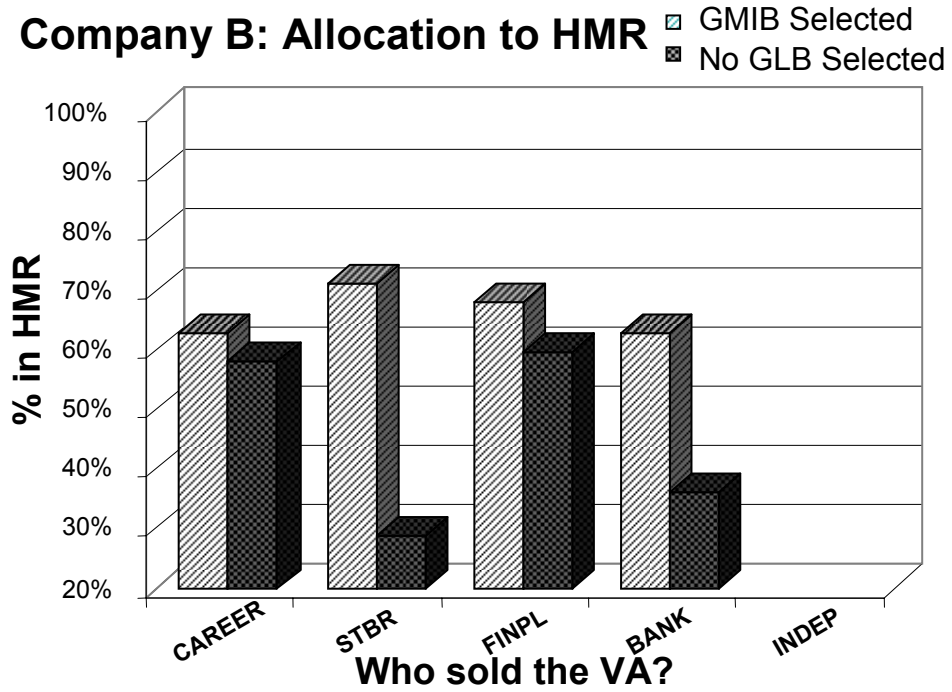
Notes: This figures displays the percent of the variable annuity sub-account within the policy sold by Company A and B – two anonymous companies -- that are allocated to high and medium risk (HMR) asset classes, when the GMIB is selected and when no Guaranteed Living Benefit (GLB RIDER) is selected.

Figure #2: Distribution channel and the impact on risky allocation.



Notes: Variable annuities manufactured by company A can be purchased thru career insurance agents, stock brokers (wirehouses), financial planners and banks. This chart illustrates the impact of this distribution channel on the allocation to HMR assets, both with and without guarantees.

Figure #3: Another example, impact of distribution channel on risky allocation.



Notes: Variable annuities manufactured by (anonymous) company B can be purchased thru career insurance agents, stock brokers (wirehouses), financial planners and banks. This chart illustrates the impact of this distribution channel on the allocation to HMR assets, both with and without guarantees.

Table #1:

AGE	Company A			Company B		
	% in HMR		Additional HMR Exposure	% in HMR		Additional HMR Exposure
	GMIB only	no GLB RIDER		GMIB only	no GLB RIDER	
<=40	86.90%	76.30%	10.60%	67.80%	62.80%	5.00%
41-45	85.30%	73.10%	12.20%	67.80%	62.20%	5.60%
46-50	84.60%	70.60%	14.00%	67.60%	61.70%	5.90%
51-55	83.80%	66.70%	17.10%	67.90%	59.90%	8.00%
56-60	82.40%	62.20%	20.30%	67.60%	54.90%	12.70%
61-65	82.00%	59.00%	23.00%	67.30%	51.10%	16.10%
66-70	82.10%	58.30%	23.80%	67.10%	50.00%	17.20%
71-75	81.60%	59.30%	22.20%	66.80%	48.70%	18.10%
76-80	79.70%	61.90%	17.80%	67.60%	51.10%	16.40%
>80	77.50%	46.70%	30.80%	N/A	46.40%	N/A
Total # of Policies	104,377	66,085		89,949	36,169	

Notes: This table presents the percentage invested in high and medium risk (HMR) assets for companies A and B for ten age groups in five year buckets. Funds regarded as high or medium risk: large-cap, mid-cap, and small-cap stock funds, high yield bond fund, balanced fund, specialty/sector fund, international equity fund. The rest of available fund classes - investment-grade bond fund, money-market fund, and fixed fund - are considered as low risk assets. The first column for each company represents percentage allocation to HMR for investors who selected a Guaranteed Minimum Income Benefit (GMIB); the second column represents allocation to HMR for those who did not select any guaranteed living benefit (GLB RIDER). Depending on the particular age group, VA policyholders have an increased allocation to HMR asset classes when they select a GMIB. For company A this ranges from 10% to 30%. For company B this ranges from 5% to 16%. Note that in the case of company B, contracts were not issued above the age of 80. Also, HMR allocations are not directly comparable across companies due to different investment options.

Table 2

Linear regression (pooled sample of seven companies). Dependent variable - percentage of the VA account invested in HMR assets

Variable	With GMIB		Without GLB RIDER	
	Estimate	t-value	Estimate	t-value
Intercept	0.9132*	59.22	0.9921*	131.92
LOGACC	-0.0018*	-5.66	-0.0178*	-30.42
MALE	0.0123*	16.45	0.0256*	18.24
AGE	-0.0015*	-43.09	-0.0041*	-76.1
AGE2	1.8E-06*	5.57	-3.5E-06*	-8.02
IRA	-0.0044*	-5.59	-0.0098*	-6.37
CAREER	-0.0388*	-8.8	0.0757*	18.82
STBR	0.0112*	2.6	0.0296*	7.27
FINPL	0.0025	0.57	0.1474*	31.75
BANK	-0.0576*	-13.02	-0.1433*	-34.42
C1	-0.1331*	-9.26	-0.0928*	-23.29
C2	0.0383*	2.67	0.0808*	21.68
C3	0.0883*	6.14	0.2013*	38.11
C7	0.0285*	1.98	0.1654*	44.29
C8	-0.0489*	-3.33	-0.0450*	-9.91
C9	-0.0733*	-5.03	0.2315*	48.5
C10	0.1063*	7.38	0.2058*	47.94
Adjusted R ²	0.1541		0.159	
N. of observations	368,005		272,564	

Notes: The above table presents the results of the least squares regressions with the percentage invested in the high and medium risk assets as the dependent variable. The first regression is based on the sub-sample of variable annuity policies where no guaranteed living benefits (GLB RIDER) was selected. The second regression is based on the sub-sample where longevity-put was selected. This regression is based on a pooled sample of all ten companies. We controlled for the various companies using seven dummy variables, which represent the seven companies with significant number of policies where GMIB was selected. The omitted category is the other three companies.

* Significant at 5% level

Table 3: Linear regression (companies A and B). Dependent variable - percentage of the VA account invested in HMR assets

Variable	Company A				Company B			
	With GMIB		Without GLB RIDER		With GMIB		Without GLB RIDER	
	Estimate	t-value	Estimate	t-value	Estimate	t-value	Estimate	t-value
Intercept	0.6388*	83.56	0.5129*	28.07	0.9483*	111.09	0.9852*	60.72
LOGACC	0.0030*	5.12	0.0052*	3.71	-0.0067*	-10.24	-0.0219*	-16.46
MALE	0.0036*	2.76	0.0037	1.15	0.0203*	13.28	0.0364*	12.18
AGE	-0.0007*	-11.21	-0.0026*	-20.62	-0.0018*	-24.03	-0.0045*	-37.13
AGE2	1.7E-06*	2.79	-9.1E-06*	-8.7	3.8E-06*	5.69	-9.3E-06*	-10.1
IRA	-0.0045*	-3.14	0.0079*	2.1	-0.0051*	-3.09	0.0135*	4.14
CAREER	-0.0022	-0.7	0.1780*	22.99	0.0268*	12.09	0.2169*	56.23
STBR	0.0778*	22.29	-0.0587*	-6.94	0.0689*	27.83	0.2386*	58.29
FINPL	0.0512*	18.67	0.2359*	31.98	0.0693*	27.84	0.3102*	63.13
Adjusted R ²	0.0174		0.1213		0.0184		0.1179	
N. of observations	89,947		36,165		103,348		65,855	

Notes: The above table presents the results of the least squares regressions with the percentage invested in the high and medium risk assets as the dependent variable for (the largest) two companies separately. The first regression for each company is based on the sub-sample of variable annuity policies where no guaranteed living benefits (GLB) were selected. The second regression is based on the sub-sample from the same company where longevity-put was selected. Some of the coefficient signs change based on the specific company, but notice that the age variable is always negative (with and without the GMIB) and the gender variable is positive (with and without the GMIB).

*Significant at 5% level

Table #4: Optimal Allocations to GMIB option.

% in HMR without GMIB α^*	Implied Risk Aversion γ	% in HMR with GMIB α^{**}	CE with GMIB $U^{-1}(U^{**})$	CE w/o GMIB $U^{-1}(U^*)$
GMIB Annuity Factor $g_{70} = 15$				
30%	6.67	100%	230%	140%
40%	5.00	100%	250%	170%
50%	4.00	100%	260%	210%
GMIB Annuity Factor $g_{70} = 20$				
30%	6.67	40%-50%	200%	200%
40%	5.00	70%-100%	220%	200%
50%	4.00	100%	240%	210%
GMIB Annuity Factor $g_{70} = 25$				
30%	6.67	35%-40%	200%	200%
40%	5.00	50%-60%	210%	210%
50%	4.00	70%-100%	230%	220%

Notes: The above table illustrates the change in optimal asset allocation when a GMIB is offered on a variable annuity account, as a function of the GMIB annuity factor. For example, an investor with a 40% allocation to risky equity (defined as HMR) reveals a coefficient of relative risk aversion of 5. If this individual is offered a GMIB option at the price of \$20 per dollar of lifetime income, he will change his allocation to something between 70% and 100% HMR because of the downside protection. This investor will also experience an increase in certainty equivalent (CE) utility from 200% of initial wealth to 220% of initial wealth. In other words this particular GMIB is welfare enhancing. The underlying parameters were calibrated to fit the historical risk and return parameters of the variable annuity sub-accounts.