Longevity Risk and Life Annuities:

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Abstract

Longevity risk is faced by individuals, corporations as well as society and their governments. In this brief article I review the various dimensions and aspects of longevity risk as they relate to the market for life annuities and mortality-contingent claims. My main argument is that regardless of one's perspective on longevity risk, this field will continue to provide fertile ground for advanced research for many, albeit uncertain, years to come.

KEYWORDS: Pensions, Retirement, Mortality, Aging, Insurance, Portfolio Choice, Asset Allocation.

1 Introduction: What is Longevity Risk?

According to the Merriam-Webster dictionary, the definition of the word *longevity* is "a long duration or length of individual life" and the word *risk* in the broadest sense of the term is defined as "something that creates or suggests a hazard". Placing these two definitions side by side, implies that a long duration for individual life creates a hazard, which might seem odd to anyone not trained in actuarial science or insurance. After all, some might wonder, how can a long life be hazardous?

Indeed, although most readers would agree that a long life should be a blessing, it can be a costly one. From an individual perspective, financing 90, 95 or even 100 years of life can be expensive, especially if it is unexpected. This is at the heart of longevity risk, which is related to the cost side of a long life and is the exact opposite of mortality risk, which is the economic cost to survivors and family of an unexpectedly short life.

From a financial point of view the practical implications of longevity risk come from its uncertainty as opposed to its length. If everyone knew exactly how long they would live – even if this was a very long time – longevity risk would be non existent and, I claim, there would be very little interest in, and need for, a market for life annuities or general mortality-contingent claims. The greater the uncertainty around the length of human life, both individually and in aggregate, the greater is the need for a well developed annuity market. This relatively trivial statement – that the variance is more important than the mean – has critical implications for the future of the insurance industry as it relates to the provision of retirement income, and I will get to this a bit later.

On a more formal level, if the symbol $\mathbf{T}_x^z(i) > 0$ denotes the remaining lifetime random variable for individual i,aged x in the year z (for example $x = 65$ in the year $z = 2006$), then obviously $E[\mathbf{T}_0^z]$ denotes general life expectancy at birth in year z. Of critical importance to the topic of longevity risk is the strong disagreement amongst demographers as to whether there is a biological upper bound on $E[\mathbf{T}_0^z]$ as z goes to 2010, 2020 and beyond.

Well-cited researchers such as Olshansky and Carnes (2001) claim that life expectancy at birth is not likely to ever exceed the age of 85 or 90. These rather pessimistic authors are amongst a group of scholars who believe that the remarkable reductions in mortality rates across all ages during the last two centuries are a "disconnected sequence of unrepeatable revolutions" and that biological barriers create a fixed upper bound on life expectancy.

Others, such as Oeppen and Vaupel (2002) take a diametrically opposing view and argue that there are no biological limits to life expectancy. They offer as evidence the fact that female life expectancy in record-holding countries has been increasing at a steady pace of 3 months per year for the last 160 years. Indeed, when (record) life expectancy in any given year is regressed against the last 160 years of data, the trend is remarkably linear with an R^2 value of 99% according to Oeppen and Vaupel (2002). These authors are extremely optimistic that biomedical research will yield unprecedented increases in survival rates.

It is precisely this disagreement amongst experts and the variance of professional opinion that leads to the richness and fertility of longevity risk as a research field. Likewise, in the language of financial economics there should be a market price for the undiversifiable portion of longevity risk. I will get to this aspect in a moment.

Thus, in this article I will focus on three distinct aspects or dimensions of longevity risk. The first dimension deals with longevity risks' relation to individuals and their personal financial decisions. It addresses the question: How should individuals manage longevity risk? The second dimension of this article relates to the role of intermediaries and insurance corporations that take-on and absorb longevity risk. How can they measure and then manage this risk effectively? The final dimension relates to the role of governments and society as a whole in managing longevity risk. Of course, given the strict limits on time and space I will be brief with the usual academic citations and literary references, but will make sure to highlight some key papers in all three (individual, business and government) dimensions.

I also refer the interested reader to Milevsky (2006) for a much more extensive list of relevant and important references related to the market for life annuities. In addition, the December 2006 issue of the *Journal of Risk and Insurance* is devoted entirely to the topic of longevity risk and I encourage the interested reader to consult there as well.

2 Individual Dimension: Why Buy Life Annuities?

One of the stylized facts in the research field of longevity risk and life annuities is the widespread reluctance of individuals to voluntarily purchase life annuities. Life annuities hedge against longevity risk. Modigiliani (1986) in his Nobel prize lecture referred to this phenomena as the "annuity puzzle" and many papers have followed since. Indeed, in the U.S. less than 5% of the population of retirees has voluntarily purchased a life annuity, according to a surveys conducted by the consulting firm LIMRA International. In the U.K. recent changes to pension law abolished the mandatory conversion of certain tax-sheltered savings plans to life annuities at the age of 75, mostly because of public dislike and distrust of annuitization.

Therefore, to understand why life annuities are so unpopular – and the implications of this to the management of personal longevity risk – we must start at the micro economic foundations of the demand for life annuities.

I now provide a simple two-period micro-economic example that illustrates the gains in personal utility from having access to a life annuity market. Assume you have an initial sum of \$1 which must be allocated and consumed during either of the next two periods. The consumption which you must select, denoted by C_1 and C_2, takes place for convenience at the end of the period. There is a p_1 probability that you will survive to (consume at) the end of the first period, and a p_2 probability of surviving to (consuming at) the end of the second period. The periodic interest rate – which is also the return on investment in this simple model – is denoted by R.

There is some disagreement as to whether individuals are able to formulate accurate and consistent probability estimates for their own survival rates $(p_1, p_2.)$ For example Smith, Taylor and Sloan (2001) claim that in general individual's estimates are consistent with objective mortality tables, and this view is echoed in Hurd and McGarry (1995) as well. In contrast, Bhattacharya, Goldman and Sood (2003) claim that individuals have systematic biases in the way they perceive their own mortality rates. This is clearly an ongoing area of research.

Either way, assume that you know your p's and your objective is to maximize your discounted utility of consumption. To make things even easier, assume you have logarithmic preferences for consumption utility. This is all standard. In the absence of life annuities or any market for life contingent claims, your objective function and budget constraint would be described by:

$$\max_{\{C_1,C_2\}} \quad \text{Utility} = \frac{p_1}{1+\rho}\ln[C_1] + \frac{p_2}{(1+\rho)^2}\ln[C_2], \tag{1}$$

$$\text{st} \qquad 1 = \frac{C_1}{1+R} + \frac{C_2}{(1+R)^2}, \tag{2}$$

where ρ is your subjective discount rate. This "toy model" is a subset of a classical lifecycle model of investment and consumptions which is at the heart of microeconomics and is described in greater length in most textbooks. Solving the problem one obtains the following optimal values for consumption over the two periods. Namely;

$$C_1^* = \frac{p_1(\rho R + R + \rho + 1)}{p_2 + p_1\rho + p_1}, \qquad C_2^* = \frac{p_2(1 + 2R + R^2)}{p_2 + p_1\rho + p_1} \tag{3}$$

The ratio of optimal consumption between period one and period two, is: $C_1^*/C_2^* = p_1(1+\rho)/p_2(1+R)$. Now we arrive at the important qualitative implication.

When the subjective discount rate is equal to the interest rate ($\rho = R$), then $C_1^*/C_2^* = p_1/p_2$, which is the ratio of the survival probabilities, which is strictly less than one. Stated differently, the optimizer will consume less at higher ages. In fact, you might rationally starve to death! This insight regarding optimal consumption over the lifecycle is quite robust and explained in Modigliani (1986), amongst other references that extend this from two periods to continuous time. If you can not insure of hedge your longevity risk, which is the probability of living for two (as opposed to just one) period, then you will adapt to this risk by taking your chances and consuming less as you age. As you will see in a moment, there are alternative strategies that provide greater utility and greater consumption possibilities.

In the presence of actuarially fair life annuities – which hedge against personal longevity risk – optimal behavior is quite different. If you are given the opportunity (i.e. access to a market) to hedge your longevity risk, you should do so. The budget constraint in equation (2) will change to reflect the probability-adjusted discount factor. The optimization problem is now:

$$\max_{\{C_1,C_2\}} \quad \text{Utility} = \frac{p_1}{1+\rho}\ln[C_1] + \frac{p_2}{(1+\rho)^2}\ln[C_2], \tag{4}$$

$$\text{st} \qquad 1 = \frac{p_1 C_1}{1+R} + \frac{p_2 C_2}{(1+R)^2}, \tag{5}$$

Notice the only difference between this and the previous optimization problem is the constraint itself. If you can hedge personal longevity risk in the insurance market, the present value of any given consumption plan will be lower. It is cheaper to finance the same standard of living if your assets are lost at death.

Yes, it is questionable whether one should use the same survival probability (p_1, p_2) in the objective function and the constraint do to adverse selection issues in the annuity market. I refer the interested reader to the paper by Finkelstein and Poterba (2002) for a discussion of this issue. Nevertheless, in this case the optimal consumption is denoted by C_1^{**}, C_2^{**}, and is equal to:

$$C_1^{**} = \frac{\rho R + R + \rho + 1}{p_2 + p_1\rho + p_1}, \qquad C_2^{**} = \frac{1 + 2R + R^2}{p_2 + p_1\rho + p_1} \tag{6}$$

The important point to notice is that $C_1^{**} = C_1^*/p_1$ and $C_2^{**} = C_2^*/p_2$, which implies that the optimal consumption is greater in both periods, when one has access to life annuities regardless of whether adverse selection plays a significant role. Specifically, at time zero, the individual would purchase a life annuity that pays C_1^{**} at time 1 and C_2^{**} at time 2. The present value of the two life annuities – as per the budget constraint – is one dollar. In this case, the ratio of consumption between period one and period two, is: $C_1^*/C_2^* = (1+\rho)/(1+R)$. When the subjective discount rate is equal to the interest rate ($\rho = R$), then $C_1^*/C_2^* = 1$.

The point in all of this is as follows algebra is as follows. Buying a life annuity allows rational consumers to eliminate or hedge their personal longevity risk. Of course, in the absence of life annuities, the best one can hope for is to allocate investment assets in a way that minimizes the lifetime probability of ruin, which is an approach taken by Young (2004). But this is also why Yaari (1965), Richards (1975) and more recently Ameriks, Veres and Warshawsky (2001) advocate that individuals should allocate a substantial fraction of their retirement wealth to life annuities. In fact, Yaari (1965) advocates complete annuitization at any age in the absence of bequest motives. A recent paper by Milevsky and Young (2006) looks at annuitization as an optimal timing problem, but all of these papers are based on the same idea. Annuitization hedges against longevity risk.

3 Business Dimension: Can Companies Hedge this Risk?

When individuals decide to hedge or insure their own longevity risk by transferring their personal exposure to intermediaries, these insurance companies then face aggregate mortality risk. This applies when selling life annuities or any other form of lifetime income such as guaranteed minimum withdrawal benefits (GMWBs), which have become very popular in the U.S. retirement market. Sometime longevity risk is interlinked with market and interest risk as in the case of guarantee annuity options (GOAs).

It is commonly thought that insurance companies (or their own re-insurers) can completely eliminate mortality and longevity risk by selling enough of claims to diversify away mortality. The law of large numbers is often invoked as a justification. But, the truth is far more complicated. Indeed, if there is systematic uncertainty regarding aggregate mortality trends – for example the research cited earlier by Olshansky and Carnes (2001) versus Oeppen and Vaupel (2002) – then selling more annuities and longevity-related insurance will not necessarily eliminate idiosyncractic risk. The following example should help explain the impact of the stochasticity of aggregate mortality.

Assume that an insurance company sells a one-period longevity insurance policy which pays \$2 if the buyer survives to the end of the period, but pays nothing if the buyer dies prior. Assume the insurance company issues N of these policies – at a price of \$1 per policy – to N independent lives, each of whom has an identical probability p of surviving and probability $(1-p)$ of dying prior to the payout date. Each longevity insurance policy generates an end-of-period Bernoulli liability for the insurance company. The random variable w_i can take on a value \$2 with probability p and a value of \$0 with probability $(1-p)$. The expected value $E[w_i] = 2p$ and the variance is $var[w_i] = p(2-2p)^2 + (1-p)(0-2p)^2 = 4p(1-p)$. In the simple case that $p = 0.5$, this collapses to $E[w_i] = 1$ and $var[w_i] = 1$ as well as $SD[w_i] = 1$.

Let the random variable, $W_N = \sum_{i=0}^{N} w_i$ denote the insurance company's aggregate liability at the end of the period, from selling a portfolio of longevity insurance policies. These independent longevity insurance exposures leave the issuing company faces an aggregate expected payout of $E[W_N] = N2p$, an aggregate variance of $var[W_N] = N4p(1-p)$ and an aggregate standard deviation of $SD[W_N] = 2\sqrt{Np(1-p)}$. The i.i.d. nature of the exposures allows me to add-up the individual variances. This immediately leads us to the well-known results that the standard deviation per policy goes to zero in the limit as $N \to \infty$. Stated technically:

$$\lim_{N\to\infty} \frac{1}{N} SD[W_N] = \lim_{N\to\infty} 2\frac{\sqrt{p(1-p)}}{\sqrt{N}} \to 0 \tag{7}$$

This is a special case of the law of large numbers (LLN) which states that $\lim_{N\to\infty} \frac{1}{N} W_N \to$

p.If the insurance company sells enough of these longevity insurance policies their risk exposure (per policy) goes to zero. Yet another interpretation of this statement is that longevity risk is completely diversifiable and not compensated by markets in equilibrium when claims are i.i.d.

What happens when the probability parameter p is unknown? This is a critical aspect of longevity risk. It is an aggregate longevity risk as opposed to a personal longevity risk. This is also equivalent to not knowing the hazard rate or instantaneous force of mortality underlying the probabilities, modeled in various recent papers such as Schrager (2006), or Denuit and Dhaene (2006). Stated differently, what if you have an estimate of versus a certainty for p?

The underlying payoff function in my simple example is now defined as $w_i^* = 2$ with probability $\widetilde{p}$ and $w_i^* = 0$ with probability $(1 - \widetilde{p})$. The asterisk on top of the w_i^* reminds the reader that the parameter $\widetilde{p}$ itself has its own (symmetric) distribution, which I assume takes on a value of $p + \pi$ with probability 1/2 and a value of $p - \pi$, with a probability of 1/2.

Obviously I must impose a restriction on the newly defined uncertainty parameter π, namely that $\pi \leq 1 - p$ and $p > \pi$. Also, by definition $E[\widetilde{p}] = 0.5(p + \pi) + 0.5(p - \pi) = p$. For example, one might assume that $\pi = 0.1$, $p = 0.5$ so that $\widetilde{p}$ takes on values of either 0.6 or 0.4. The intuitive interpretation for this would be that while the expected value $E[\widetilde{p}] = 0.5$ of the survival probability is 0.5, it is equally likely to take on a value of 0.4 (an improvement in mortality) or 0.6 (a worsening mortality).

Now define the total (aggregate) exposure of the insurance company by the notation $w_N^* = \sum_{i=1}^{N} w_i^*$, with the immediate implication that $E[W_N^*] = N2p$, which is identical to the traditional (deterministic) case. The key difference between W_N (deterministic mortality) and W_N^* (stochastic mortality) lies in the term for the variance. You are no longer entitled to add-up the individual variance terms due to the implicit dependence created by the common $\widetilde{p}$ factor. In this case – and one might argue, in reality – they are not i.i.d. claims. I refer the interested reader to Milevsky, Promislow and Young (2006) for an extensive elaborartion on this point, in which it is also shown that:

$$var[W_N^*] = 4Np(1 - p) + 4N\pi^2(N - 1). \tag{8}$$

This collapses to the familiar and intuitive $4Np(1 - p)$ when $\pi = 0$, and we are back to the deterministic mortality world. Note also that when $N = 1$, the variance of the payout is the same $4Np(1-p)$ it would be under the deterministic case, which means that an individual policy isn't any riskier under a stochastic $\widetilde{p}$ versus a deterministic $p = E[\widetilde{p}]$. It is the portfolio aggregation that creates the extra risk.

For example, if an insurance company sells $N = 10,000$ longevity insurance (a.k.a. term life annuity) policies under the parameter set $\pi = 0.1$, so that $\widetilde{p}$ takes on a value of either 0.6 or 0.4 with equal probability, the variance of the aggregate payout to the insurance company becomes $0.96N + 0.04N^2$ according to equation (8). Notice the N^2 term. The variance grows non-linearly in N, which means that that standard deviation per policy $\sqrt{0.96N + 0.04N^2}/N$, will converge to a constant 0.2 instead of zero when $N \to \infty$. No matter how many longevity insurance policies the company issues, the uncertainty (risk) per policy will never be less than \$0.20 per \$1.00 of expected payoff. The risk never goes away. Diversification can only reduce risk up to a certain point. In general, we have that $\lim_{N\to\infty} \frac{1}{N} SD[W_N^*] \to 2\pi$. The standard deviation per policy (SDP) converges to the total spread (2π) in the probability (hazard) rate.

What does all of this mean to companies who are trying to manage their own longevity risk? The main take-away is as follows. There is a growing body of research that attempts to model the evolution of aggregate morality, for example the trend in $E[\mathbf{T}_0^z]$, for $z = 2006, 2007, 2008$,etc. Various stochastic processes have been proposed and calibrated starting with work by Lee and Carter (1992), Olivieri (2001), Dahl (2004), Renshaw and Haberman (2006), Carnes, Blake and Dowd (2006), as well as Biffis and Millossovich (2006). This is also related to the pricing of guaranteed annuity options, which is addressed in Boyle and Hardy (2005). All of these papers are effectively based on the same underlying argument. One does not know with certainty the evolution of the dynamics of $\mathbf{T}_x^z$ or even $E[\mathbf{T}_x^z]$.

4 Government Dimension: Should they Get Involved?

Finally, if longevity risk is faced by individuals and corporations, this obviously becomes a public policy issue as well. If individuals do not value the longevity-insurance benefits provided by life annuities and are perhaps myopic in their investment behavior, should government mandate annuitization for tax-sheltered savings plans and other forms of personal pensions? Should governments actively try to correct behavioral biases by subsidizing (entering) the annuity market and making it more appealing to individuals? What can government policy do to encourage a deeper market for annuities? Along these lines some researchers have advocated that government issue longevity or survivor bonds as a way for governments to help insurance intermediaries as well as pension funds cope with longevity risk than can not be diversified away using the law of large numbers. See for example Blake and Burrows (2001) for this line of thinking. Other policy-oriented papers include Feldstein and Ranguelova (2001) which examine the role of life annuities in a privatized social security system, or Brown and Warshawsky (2001) which examines the role of annuities within pension plans appear to favour private sectors solutions to this problem. A number of this issues are addressed in the December 2006 issue of the Journal of Risk and Insurance, and I therefore do not elaborate further.

In sum, longevity risk – regardless of how it is defined – is faced by individuals, corporations as well as society and their governments. In a brief article such as this, it is virtually impossible to review or discuss all papers within this broad field. Hopefully I have been able to give a flavour of the various dimensions and aspects of longevity risk as they relate to the market for life annuities and mortality-contingent claims. I believe this field will continue to provide fertile ground for advanced research for many, albeit uncertain, years to come.

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