

A Diffusive Wander over the Human Life Cycle:

How Quantitative Finance is Transforming the Field of Personal Wealth Management

By:

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When I mention the term *Ito's Lemma* or *Diffusion Processes* to our new graduate students in finance and economics – upon being asked about the tools of my research trade – they immediately assume that I work in the esoteric field of option pricing and derivative hedging. Most are surprised to learn that I actually use these concepts to analyze practical questions such as how much and which type of life insurance people should purchase, the best age at which to start drawing a retirement pension annuity or the optimal home mortgage loan for their family. Indeed, the models of quantitative finance that spawned a derivatives revolution in the 1970s and 1980s, which then filtered through to real options in corporate finance and strategy in the 1980s and 1990s have finally arrived at the laymen's doorstep. The objective is crystal clear, namely, to help individuals make better personal wealth and risk management decisions.

In fact, in the last decade financial luminaries such as Harry Markowitz, Bill Sharpe and Robert Merton have been penning articles in the nascent field of what I like to call Quantitative Wealth Management (abbreviated QWeMa). This article is intended as a brief survey of this emergent field. Please refer to the academic bibliography at the end, for a sample of some key pieces in this growing and exciting field.

There are a number of factors that have been driving the 'quantification' of personal wealth and risk management, but it seems the primary culprit has been the spiral

of wealth management products and choices that can leave even the most seasoned financial advisors dazed and confused. Add this growing menu to the fiduciary responsibility faced by many advisors who do not want to (appear to) make arbitrary and possibly biased financial decisions, and the need for expert tools and systems becomes evident.

Let us take a random walk over the human life-cycle of QWeMa.

1.) Portfolio Selection with Human Capital

While classical Markowitz theory and the ubiquitous efficient frontier predates much of modern financial economics, the application of this concept to human capital and broader wealth has only started recently. To understand human capital and its role in portfolio selection, we must step back for a moment and imagine the individual investor/consumer -- who is the *raison detre* of QWeMa -- as a small, privately held and poorly diversified corporation called **You Inc.** The following table provides a snap-shot view of the balance sheet of You Inc.

Balance Sheet of You Inc.

Assets <ul style="list-style-type: none"> • Bank Accounts • Housing • Stocks & Bonds • Car & Vehicles • Small Business Equity • PV of Pension 	Debt + Liabilities <ul style="list-style-type: none"> • Mortgages • Consumer Loans • Credit Cards • Student Loans
+ HUMAN CAPITAL	Total Personal Equity <ul style="list-style-type: none"> • True Net Worth

Note that in contrast to a FASB or GAAP accounting standards applied to publicly traded companies, we place an asset called human capital (HC) on the left-hand side of the personal balance sheet. Human capital is a measure of the present value of future wages, income and salary that will be earned over the human life cycle. Sure, you can't really touch, feel or see human capital but, like an oil reserve deep under the ocean, it will eventually be extracted, so it's definitely worth something today. More importantly, it has its own risk and return characteristics and can be modeled as a random variable that is correlated with a variety of economic factors. In fact, you can think of your human capital as a continuous-time stochastic process which is subject to unpredictable jumps and discontinuities. This process pays a lumpy stream of dividends, it eventually converges to zero – if you are still alive -- and your objective is to maximize these dividends in some risk-adjusted manner. Remember that you can control your HC process by investing in education, making prudent career choices (and eating healthy).

Now, with this preliminary framework in mind, recall the Markowitz paradigm dictates that *total* capital – both human and financial -- should be properly diversified. Thus, in the early stages of the life-cycle, financial capital and investments should be used to hedge and diversify human capital. Think of investable assets as a defense against adverse salaries and wages as opposed to an isolated pot of money that has to be allocated.

To understand the implications of modeling human capital as a random variable, let's take an example. For a tenured University professor like myself, human capital -- and the subsequent defined benefit pension I am entitled to – has the risk and return properties of a fixed-income bond fund paying monthly coupons. In a sense, I am a walking inflation-adjusted real return bond. Therefore, I have very little need for fixed-income bonds, money market funds and other guaranteed products in my discretionary financial portfolio. In fact, my personal savings are quite heavily invested in individual equities and broad-based investment funds. The opposite advice would be relevant for the MBA and graduate students I teach, who intend to work in the financial services sector when they graduate.

Thus, one of the fertile areas of research in QWeMA revolves around how to construct a financial portfolio – and where to invest your personal savings – so that your total portfolio is properly diversified. This line of research examines the wage profile of various career occupations and then models their co-variation with general financial markets to arrive at tailor-made investment solutions based on unique idiosyncratic factors.

Yet, QWeMa goes far beyond tailoring personalized investment portfolios for dentists, lawyers and doctors. The ideas can be used to examine and think about a number of other issues.

2.) Asset Location versus Asset Allocation

Although income tax regulations are country specific, most jurisdictions around the world offer individual tax shelters and concessions for long-term savings in which capital gains, dividends and even interest income is tax-deferred. These products might also be associated with personal pension plans or be part of a stand-alone life insurance and saving policy. Regardless of the exact substance or form, these strategies often leave individual investors with a difficult decision regarding which assets should be subjected to which tax treatment. This is known as the asset location problem as opposed to asset allocation problem, which has spawned yet another interesting avenue of research within QWeMa. From a mathematical point of view, these multi-period optimization problems are highly path-dependent due to the peculiar tax asymmetries and they quickly escalate in dimensionality. For example, while the gains on a stock are usually taxed more favorably compared to gains on a bond, if they are placed within tax shelters and held for short periods of time the situation might be reversed if upon exiting the shelter the income is penalized in some manner. Indeed, personal income taxes can severely distort optimal trading, hedging and investment strategies; far beyond the perfect and frictionless markets envisioned by the purists.

3.) Home Mortgage Financing

Beyond investment and portfolio issues, most individuals purchase a personal residence at some point in their life, yet very few have the liquid wealth to pay for this transaction in cash. Mortgage financing is usually the way in which this leveraged positioned is accomplished. And, just like publicly traded companies face a myriad of choices between fixed and floating rate obligations in their capital structure, so too individuals must decide on the composition and characteristics of their debt. Depending on the jurisdiction and country, consumers must decide whether to go 'long' or 'short' term or whether to 'close' or leave 'open' their home mortgage. In some cases, individuals must further decide whether to borrow in real (after-inflation) or nominal terms or whether to link their payments to a given index. In all of these personal deliberations, a robust and dynamic model for the evolution of the term-structure of interest rate is required. But, the models must do more than explain the evolution of the curve over short periods of time and under the Q-measure, akin to what an option trader might need. They must also provide realistic long-term P-measure forecasts and risk metrics taking into account the option to default, prepay and move. The presence of complex penalties and prepayment charges further fuels the demand for rigorous and sound advice.

4.) Life and Health Insurance

Personal risk management is an exercise in protecting human as well as financial capital. Recall that both forms of capital should diversify each other. So too property insurance should be viewed as a hedge against the loss of financial capital and life insurance should be treated as a hedge against the unsupportable loss – to the surviving family -- of human capital. Thus, all forms of life, health and disability insurance shouldn't be acquired as an investment, rather they are purchased for their hedging properties. If something happens to human capital, the insurance will payoff, but if nothing (bad) happens to human capital, the insurance will perform poorly. Insurance is an asset class with a negative expected rate of return (on a pre-tax basis, at least) but nevertheless forms part of the optimal portfolio because of the negative correlation matrix. The concept might

sound simple but the broad menu of insurance choices available – such as term life, whole life and universal life, to name a few – generates yet another difficult portfolio selection problem. In many cases, insurance products provide guaranteed renewability and contain implicit options on mortality tables. Adverse selection, which is at the heart of insurance pricing, becomes an optimal strategy in its own right.

5.) Employee Compensation and Proper Incentives

QWeMa can also lend insights into the optimal design of employment compensation contracts. The trend towards company stock-linked compensation and incentive stock options creates a need for normative advice on optimal exercise strategies in the presence of highly illiquid and concentrated portfolios. Furthermore, if granting more options to a greater number of employees only serves to increase their risk profile, the incentive effects might be counter-productive. A sound mathematical model of the tradeoff between the various risk factors is required for a careful analysis. Indeed, the recent accounting debate surrounding the *cost* of an incentive stock option must be prefaced on a solid understanding of how individuals *value* these illiquid instruments.

6.) Choice of Defined Benefit vs. Defined Contribution Pensions

For many years, the venerable defined benefit (DB) pension plan was the mainstay of retirement pension provision. An individual worked for the same company during his or her entire life, and earned a pension annuity based on career average earnings, and/or the number of years of credited service. In DB plans the ‘risk’ was borne by the employer who had to provide and support the guarantee. But over the last ten to fifteen years there has been a sharp and noticeable shift towards defined contribution (DC) pension provisions around the world. Under these plans, the employee is entitled to fixed periodic contribution – with full control over its allocation – but where the risk is squarely in the hands of the employee as well. And, although it is debatable what is best for individuals at which point in their life, the transition process from DB to DC often involves a number of

choices that lend themselves to mathematical modeling. For example, what is the value of the option to choose which plan to belong to? Or what is the optimal mix between DB and DC plans from a portfolio perspective? Actuaries have only recently started to embrace the preaching's of financial economics as they pertain to the valuation of pension benefits and QWeMa models are intrinsic to the analysis.

7.) Starting a Retirement Pension Annuity

The decision of when to retire can lead to some fascinating and complicated optimal stopping problem. On the one hand we have the illiquidity and irreversibility of a pension annuity which would lead people to delay annuitization. On the other hand, we are faced with lifetime uncertainty and the desire to hedge against longevity risk. A number of recent models have been developed to help consumers understand and appreciate these tradeoffs. Once again, we must turn to a holistic risk and wealth management framework to provide proper guidance.

8.) Estate Planning and Tax Arbitrage.

Many countries impose estate and wealth taxes on individuals upon their demise. And while death and taxes are truly unavoidable, the ability to minimize the impact of the former on the latter is a rich and fertile endeavor for many lawyers and accountants specializing in estate transfers. Interestingly, a number of provisions within the tax code allow for mortality and tax arbitrage by allowing portions of the estate to escape taxation. The precise construction and implementation of these strategies often comes down to satisfying certain internal rate of return criteria or probabilistic thresholds that, once again, fall naturally within the domain of QWeMa. (Can a course called *Ito Calculus for Attorneys* be far behind?)

9.) The Real Options in Your Life

From a broader perspective, many of the above-mentioned optimization problems involve an element of timing and irreversibility, both of which bring to mind the classical call and put option analogy. Recall that an American option differs from its European counterpart in that one can choose when to exercise the option. Taking this concept further, we can frame many decisions – financial as well as not financial ones -- that people face over their life, as option pricing problems. Viewed from this perspective, the optimal career, time to purchase a house and even age at which to get married, have children and get divorced all become an exercise in American option pricing, or at the very least a dynamic programming problem. Time constraints prevent an in-depth elaboration, but one can imagine the numerous additional applications.

Indeed, one of the defining features of QWeMa is that, in contrast to the recent work in behavioral financial economics which sheds light on the psychological biases that lead to consistent human mistakes, we are taking a normative perspective and trying to help people make better decisions. In sum, the field of Quantitative Wealth Management likely has its best years ahead, both in terms of academic developments and the application by industry.

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