Defined Benefit to Defined Contribution and Back: Valuation of the Florida Pension Election

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Abstract

During the year 2002, The State of Florida's 600,000 public employees were given the choice of converting their traditional Defined Benefit (DB) pension plan into an individual-account Defined Contribution (DC) plan with full control over asset allocation and investment decisions. To mitigate some of the risk and uncertainty in the decision, the State granted each employee electing the DC plan an additional option to switch back into the DB plan at any point prior to retirement. This option has been labeled the *2nd election* by the State and the cost of re-entry is fixed at the accumulated benefit obligation (ABO) of their pension entitlement.

Our paper presents some original analytic insights relating to the optimal time and financial value of this unique 2nd election. We start with a simple deterministic model to provide intuition and conclude with a stochastic model that derives a formal upper bound for the economic value. The conclusions from our analysis differ from the results of Lachance, Mitchell and Smetters (*JRI*, 2003). We argue that the 2nd election behaves less like a traditional downside-protected put option and more like a linear forward contract. We estimate that the value of this 2nd election is at most 30% of the DC contribution rate and only when exercised at the optimal time. Furthermore, for most State employees above the age of 45, the 2nd election has little economic value since the DB plan dominates the DC plan from day one. Of course, it remains to be seen what percent of Florida's 600,000 employees will elect to behave rationally with their newfound pension autonomy.

1 The Florida Pension Election:

During the year 2002, The State of Florida's 600,000 public employees were given the choice of converting their traditional Defined Benefit (DB) pension plan into an individual-account Defined Contribution (DC) plan with full control over asset allocation and investment decisions. This new Public Employee Optional Retirement Program (PEORP) has been the focus of intense scrutiny by local and national media because it is the largest such pension conversion in the history of the U.S. and is being viewed by some observers as a potential laboratory for Social Security reform.

Interestingly, to mitigate some of the risk associated with this decision, the State granted each employee electing the DC plan an option to switch back into the DB plan at any point prior to retirement. This option has been called the *2nd election* by the State authorities and we will adopt this name. The cost of getting back into the DB plan is the accumulated benefit obligation (ABO) of their pension entitlement. The ABO is effectively the present value of that portion of the life annuity (pension) to be received at retirement, based on the number of years of service and salary at the time of computation. For future employees – i.e. those not in the plan at the time the PEORP was initiated – the buy back price will be the accumulated actuarial liability (AAL).

Our paper presents some original analytic insights relating to the optimal time and financial value of this unique 2nd election. We start with a simple deterministic model to provide intuition and conclude with a stochastic model that derives a formal upper bound for the economic value.

We are careful to distinguish between the financial economic value of the 2nd election – which is the focus of this paper – versus the more vague and controversial pension 'funding cost' of providing the 2nd election to the employee. While the former is related to portfolio replication and dynamic hedging of guarantees, the latter depends on various actuarial standards of practice, assumptions and cost methods that are beyond the scope (and interest) of this analysis. We refer the interested reader to the work by Haberman and Sung (1994) as well as O'Brien (1986) for stochastic models of pension plans that are focused on actuarial funding methods.

Most importantly, the conclusions from our analysis differ from the results of Lachance, Mitchell and Smetters (2003) which were recently published in the *Journal of Risk and Insurance*. We argue that the guarantee embedded within the 2nd election is not really a traditional downside-protected put option. If the derivative analogy can be used at all, then Florida's 2nd election is closer to a linear forward contract. We estimate that the value of the 2nd election is, at most, worth 30% of the DC contribution rate and only when exercised at the optimal time. Furthermore, for most State employees above the age of 45, the 2nd election has little economic value since the DB plan dominates the DC plan from day one. To be clear, we are not saying that Floridian's above the age of 45 should not be opting into the State's DC plan. Rather, for those who intend to retire (with 100% certainty) from the State plan, the DB choice is optimal since they will accumulate greater retirement wealth. *And, if these employees behave sub-optimaly relative to our optimal exercise rule, the State of Florida can only save money.*

On a conceptual level, our paper goes beyond some of the recent academic literature on pension choices and guarantees – for example Sherris (1995), Pennacchi (1999) and Lachance, Mitchell and Smetters (2002), as well as Childs, Fore, Ott, and Lilly (2002) – by providing simple analytic guidance on the optimal choice between DB and DC plans. We make some innocuous and simplifying assumptions – that can be changed from liberal to conservative depending on the need – in order to shed intuitive light on the decision.

>From a broader academic literature perspective, this paper attempts to bring the tools of financial economics and No Arbitrage valuation to the field of pensions, retirement planning and longevity insurance. This is in the same spirit as recent papers by Boyle and Hardy (2002) as well as Sundaresan and Zapatero (1997).

1.1 Agenda for the paper:

The next section develops a complete intuition for the deterministic case where interest rates, investment returns, retirement dates and employment horizons are known with perfect certainty. We then move to the stochastic case where the linearity of the payoff structure collapses the problem to the results applicable under the deterministic case. The subsequent section provides some numerical examples and actual data from the State of Florida. We refer the reader to the work by Bodie (1990) as well as Bodie, Marcus and Merton (1988) for a refresher on the basic differences between defined contribution and defined benefit pension plans. Technical proofs are relegated to an appendix.

2 Deterministic Case:

We start by assuming that all economic parameters are known with certainty. We let C_t denote the value of the Defined Contribution (DC) account at time t, and let B_t denote the value of the accumulated benefit obligation (ABO) – or the accumulated actuarial liability[1] (AAL) – in the Defined Benefit (DB) plan, at time t. While the ABO and AAL are distinct quantities, most of our comments and model apply to either metric (buy back price).

Our underlying framework revolves around a participant currently aged$-y$, with τ years of service, that has just been given the choice at time $t = 0$, (upon launching the PEORP) to move to the DC plan. The participant is also granted a 2nd election to buy back into the DB plan – if they actually left the DB plan to begin with – at any future switching time s, where $s \leq T$, which is the retirement horizon. Our main focus of analysis is two-fold. *Should the participant make the first election to switch into the DB plan? And if so, when should they elect – if ever – to switch back?*

Exhibit #1 provides a graphical illustration of the various decision points. The participant starts in the DB plan when the PEORP is inaugurated and must decide whether to switch into the DC plan (today). Assuming the participant does decide to elect the DC plan, he or she has the opportunity to make the *2nd election* and return to the DB plan anytime prior to retirement. If, on the other hand, the participant decided not to elect the DC plan at inception of the program, they have a second (and last) chance to switch to the DC plan prior to retirement.

There is an alternative choice – that we do not explicitly address here – in which the employee is given the option to leave (or freeze) the vested benefits in the DB plan, and direct new contributions only, into the DC plan. As we shall see later, this strategy is implicitly dominated by one of the two alternatives. In other words, either it is optimal to *switch* everything into the DC plan, or it is optimal to *keep* everything in the DB plan.

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[1] The AAL captures an average contribution rate on the part of the plan sponsor, and is therefore only loosely tied to any one individual's pension benefit.

Formally, given the availability of the 2nd election (a.k.a. buy-back option) at retirement time T, the participant will accumulate a total wealth of:

$$W_T[s] = B_T + (C_s - B_s)e^{\mu(T-s)}, \tag{1}$$

where μ denotes the instantaneous (deterministic) investment return or force of interest in the DC account. The s within the square brackets is meant to emphasize that retirement wealth at time T is a function of when exactly the switch from DC to DB takes place (if ever). The retirement wealth $W_T[s]$ is then converted into actual income by computing $W_T[s]/a_x$, where x is the retirement age and a_x is the relevant actuarial annuity factor which provides the indexed DB pension. For example, if the unisex price of a (3% or CPI indexed) \$1 of income per year for life starting at age $x = 60$, in an $\rho = 8\%$ (valuation) interest rate environment is $a_x = \$14.75$, then a participant with $\$100,000$ at time T in his or her retirement account would be entitled to a pension of $100,000/14.75 = \$6,779$ per annum in real terms.

Without any loss of generality, we assume that an optimal $s = 0$ implies that *either* the participant immediately switches back to the DB account, or, never bothers electing the DC account (PEORP Plan) to begin with. Note also that, if the 2nd election is *not* available, the participant's wealth at retirement will simply be $W_T = \max[C_T, B_T]$ since in a world of perfect determinism, the participant will select the plan with the greatest value at retirement.

Thus, our metric for the value of the 2nd election – in a deterministic setting, of course – is the proportional increase in retirement wealth that is obtained by having access to the 2nd election. In this spirit we define the percentage increase in time$-T$ retirement wealth:

$$\upsilon_T[s] := \frac{W_T[s] - \max[C_T, B_T]}{\max[C_T, B_T]} = \frac{B_T + (C_s - B_s)e^{\mu(T-s)}}{\max[C_T, B_T]} - 1, \tag{2}$$

assuming the election is made at time s. Quite naturally, if $s = T$, the value of $\upsilon_T[T] = 0$, since there is no gain from switching. We will return to the behavior of $\upsilon_T[s]$ – as a function of the various underlying parameters – later in the analysis.

An alternative representation of the value of the 2nd election is to locate the contribution rate in the DC plan that would have generated the same level of retirement wealth, assuming the 2nd election were not available.

One point worth noting is that at retirement, which is at time T and age x, we do not distinguish between accumulated wealth in the DC account and the value of the DB account. In other words, throughout this analysis we are implicitly assuming that pure economic value is all that matters. This is regardless of whether the funds are sitting in a DB or DC account. Of course participants with a very strong (psychological) preference for a lump sum benefit will subjectively value a dollar in the DC account – which does not force annuitization – much higher than a dollar in the DB account. The value of life annuities versus systematic withdrawal plans has been the subject of intense research in the academic literature, and we refer the interested reader to the recent paper by Brown and Poterba (2000) where they discuss the welfare gains from annuitization, and how intra-family risk sharing might influence the choice of particular pension (annuity) benefits. We also recommend the related papers by Benartzi and Thaler (2001, 2002) for a discussion of some of the behavioral aspects of pension choice.

Likewise, we abstract from group versus individual annuity pricing – *vis a vis* the potential anti-selection issues – by treating the actuarial factor a_x, independently of whether it originates in the DC or DB plan. These assumptions were made by Lachance, Mitchell and Smetters (2003) as well.

In the same spirit, we ignore mortality – during the employment years – as well as possible termination probabilities, disability, etc. These assumptions are justified because we are focusing on the *economic value* of the 2nd election to the rational employee who expects to work in the Florida Retirement System (FRS) for his or her entire life. Indeed, for those with a shorter employment horizon the optimal policy will differ, but the economic value of the 2nd election will be lower as well. In contrast, the actuarial funding method would take account of retirement probabilities, mortality, disability and other employment decrements. We view the economic value – which does not account for any termination probabilities – as an upper bound on the funding cost. We repeat, once again, that our optimality criteria is predicated on a participant who wants to maximize retirement wealth, as opposed to being concerned with early termination and liquidity concerns. *These real world, and risk averse concerns, would only serve to reduce the economic cost of providing the 2nd election to the employee.*

2.1 The Dynamics and Account Value of C_s and B_s

Both C_s (the funds in the individual DC account) and B_s (the present value of the DB benefit) evolve at different rates over time. The accumulated actuarial liability (AAL) and the accumulated benefits obligation (ABO) will also have completely different dynamics, and thus the optimal time to switch will depend on the relevant price to re-enter. It is a fact that the B_s value (price) for the ABO will be much lower than the B_s value (price) for the AAL. For now we will focus on the ABO case which, as we mentioned earlier, would be applicable to all participants that joined the Florida plan prior to the implementation date.

2.1.1 The Dynamics of C_s

First, we examine the Defined Contribution individual account. Let I_0 denote the salary (income) at time $t = 0$ – i.e. the current point in time at which the analysis is being conducted – and assume it grows at a (constant) rate of g per annum. The employer contributes to the DC account at a rate of $\widehat{c}$ percent of salary per annum which then translates into a nominal dollar contribution of:

$$c_s := \widehat{c} I_0 e^{gs}, \tag{3}$$

per annum. For example, if the (current) initial salary is $I_0 = \$30,000$, the growth rate is $g = 4.75\%$, and the contribution rate is $\widehat{c} = 9\%$, then in the tenth year, the employer's contribution would be approximately $c_{10} = \$4,342$, in nominal dollars. If the contributions are invested in an account earning μ per annum, the accumulated value of these contributions at time s would be:

$$\begin{aligned} C_s &= C_0 e^{\mu s} + \int_0^s c_t e^{\mu(s-t)} dt = C_0 e^{\mu s} + \int_0^s \widehat{c} I_0 e^{gt} e^{\mu(s-t)} \\ &= C_0 e^{\mu s} + \widehat{c} I_0 e^{\mu s} \int_0^s e^{(g-\mu)t} dt = C_0 e^{\mu s} + \frac{\widehat{c} I_0 (e^{gs} - e^{\mu s})}{g - \mu}, \end{aligned} \tag{4}$$

where the (constant) symbol C_0 denotes the funds that are already in the DC account – as a result of past contributions and gains – at the current time $t = 0$. In the special case where $\mu = g$, equation (4) converges to a limiting value of $(C_0 + \widehat{c} I_0 s) e^{\mu s}$.

For example, if $C_0 = 5,000$, (current market value of the DC account), $\widehat{c} = 0.09$ (annual contributions equal to 9% of salary), $I_0 = 30,000$ (current salary of \$30,000 per annum), $g = 0.0475$ (nominal salary increase of 4.75% per annum) and $\mu = 0.10$ (a 10% growth rate of investments in the DC account), then in ten years, the (projected) value in the DC account is $C_{10} = \$70,691$, and in twenty years the (projected) value is: $C_{20} = \$283,974$.

Finally, it is relatively easy to show – and will be of later use – that:

$$C'_s = c_s + \mu C_s, \tag{5}$$

where the prime symbol denotes a derivative with respect to the variable s. The rate of change in the value of the DC account, is the sum of the contributions to the DC account, c_s and the instantaneous investment growth of the DC account, μC_s.

2.1.2 The Dynamics of B_s

The evolution and economic value of the funds in the Defined Benefit account at time B_s depend on the pension accrual rate $\widehat{b}$, the number of years of service $(s + \tau)$, the salary $I_0 e^{gs}$ and the present discounted value of the annuity factor $e^{-\rho T} a_x$. The valuation discount rate ρ is clearly not the risk-free rate of interest in the economy, and is the topic of some controversy in the pension literature. In the Florida State pension plan, ρ is currently 8% which is much higher than the 5% risk-free Treasury rate prevailing at the time of writing. It appears the pension plan discounts (values) liabilities at the expected rate of return on assets, which obviously depends on the equity/bonds asset mix held by the State. This approach is driven by the Governmental Accounting Standards Board (GASB 25) and is somewhat alien to financial economic valuation techniques. Nevertheless, it appears that a large fraction of public and private pension plans in the U.S. are currently discounting their actuarial liabilities at a flat 8% – and have been doing so for quite a while. We refrain from debating this particular controversy in the paper, and refer the interested reader to the work by Gold (2000) for a discussion of the issues surrounding the valuation of liabilities in publicly sponsored pension plans.

In either event, this valuation rate ρ is a critical input in the decision process, and the actual formula relating DB values is specified by:

$$B_s = \widehat{b}(s + \tau)\left(I_0 e^{gs}\right)\left(a_x e^{-\rho(T-s)}\right) = k(s + \tau) I_0 e^{s(g+\rho)}, \tag{6}$$

where I_0 is the current salary, and where the new variable $k := \widehat{b} a_x e^{-\rho T}$, which roughly captures the accrual rate of each pension dollar, for each year of service. It has similar characteristics to the DC contribution rate parameter, $\widehat{c}$, which we will discuss later. Technically, there is a 6 year vesting period during which the employee does not own the funds in the DB account, but this does not affect the optimal behavior in any meaningful way since we can define years of service τ, in terms of pensionable years of service. Intuitively, the 6 year vesting period in the DB plan will induce employees to select the DC plan if their employment horizon is less than 6 years. However, once again, our representative agent is one that intends to remain within the Florida retirement system throughout his or her entire career – which is the costliest for the State – and thus provides an upper bound for the value of the 2nd election.

Also, we deliberately abstract from reality – in addition to the various deterministic assumptions – by assuming that the DB pension is a function of the final year of pay, as opposed to the *average* of the final five years of pay. This assumption was made in order to gain intuition for the deterministic optimality criteria. In practice, however, it could be trivially dealt with by letting I_0 represent the *average* salary as opposed to the *current* salary in equation (6).

For example, imagine a $y = 40$ year-old employee with $\tau = 7$ years of service, currently earning $I_0 = \$30,000$, which is expected to grow by $g = 4.75\%$ per annum. The employee earns $\widehat{b} = 1.6\%$ of salary for each year of service and plans to retire at age $x = 60$, which is in $T = 20$ years. In a $\rho = 8\%$ valuation environment where the annuity factor $a_{60} = 14.75$, one obtains a current ABO of $B_0 = \$10,005$. The ABO upon retirement – assuming, once again, that all relevant – constants remain the same, will be $B_{20} = \$494,284$. This retirement value of the ABO can also be obtained by simply multiplying the 27 years of service, time 1.6% times the final salary of \$77,571 times the annuity factor of 14.75.

Upon introduction of the PEORP, the above-mentioned employee would be given the choice to transfer the $B_0 = \$10,005$ into the DC plan, thus earning a rate of return of μ on the initial transfer and all future contributions. The question then becomes whether to move into the DC plan at all, as well as if-and-when to switch back.

Now, similar to the model for the evolution of the Defined Contribution account, we are interested in expressing the value of the DB account in terms of an instantaneous employer contribution and interest earnings. The value of the DB account at any time s should be the sum of the two components. We stress the *economic* nature of the contribution variable b_s since the actual funding of the DB pension plan would depend on a variety of actuarial assumptions and methods – for example, the entry age cost method – and thus would not necessarily reflect the contribution of the employer *per se*. Nevertheless, we define a new variable b_s, akin to the variable c_s, to be:

$$\begin{aligned} b_s = B'_s - \rho B_s &= kI_0e^{s(g+\rho)} + gk(s+\tau)I_0e^{s(g+\rho)} \\ &= \left(\frac{1}{s+\tau} + g\right) B_s. \end{aligned} \tag{7}$$

For any given time s, the instantaneous contribution b_s is an increasing function of the service record τ, an increasing function of the salary growth rate g, and an increasing function as one gets closer to retirement (i.e. as T decreases, since k will increase). These observation are all consistent with basic intuition in the matter.

Finally, similar to the relationship between the DC contribution amount c_s, and the value of the DC account C_s in equation (4), one can easily show that:

$$B_s = B_0e^{\rho s} + \int_0^s b_t e^{\rho(s-t)}dt, \tag{8}$$

which means that the value of the DB account (or the ABO) is the sum of all the contributions made on behalf of the participant – to date – accumulated at the valuation rate of interest ρ.

2.2 Maximizing Wealth

The objective of a (rational) participant should be to locate the *best* time s to switch, in order to maximize the value of wealth at retirement. In practice, of course, one might gain substantial utility from the non-monetary benefits of having a DC account over a promised DB pension, but we ignore those for now.

>From a purely mathematical point of view, the optimal time to switch is located by differentiating equation (1) with respect to s, and then using the first order condition of $\partial W_T[s]/\partial s = 0$, to isolate s^*. The asterisk atop of the s distinguishes the general time s, from the optimal time s^*. In theory, we could do the same with equation (2) and locate the switching time that maximizes the value of the 2nd election. The algebra is identical since the optimal time will be invariant to scaling.

In any event, elementary calculus leads to:

$$\frac{\partial W_T[s]}{\partial s} = (C'_s - B'_s)\, e^{\mu(T-s)} - \mu(C_s - B_s)e^{\mu(T-s)}. \tag{9}$$

The optimal switching point – when it is interior to the region $s \in [0, T]$ – is obtained at the point in time of s^*, for which:

$$\frac{\partial W_T[s = s^*]}{\partial s} = (C'_{s^*} - B'_{s^*})\, e^{\mu(T-s^*)} - \mu(C_{s^*} - B_{s^*})e^{\mu(T-s^*)} = 0. \tag{10}$$

This is the first order condition for optimizing the value of wealth at retirement.

Now, dividing both sides by common factors, substituting from equations (7) and (4), we get that s^* satisfies:

$$b_{s^*} - c_{s^*} = (\mu - \rho)B_{s^*} \tag{11}$$

For reasonable values of parameters there will be, at most, one solution to equation (11) and this will give a global maximum for $W_T[s]$. The precise conditions for this are given in the appendix and we will assume that these hold throughout this section. An easy-to-see special case is when $\mu < \rho + g$, which is satisfied in the numerical examples below.

Note from equation (11) that the optimal time to switch does not depend on the actual value of the DC account C_s. Some have erroneously suggested *heuristic* second election rules, based on the size of the account. For example, some have argued that a good candidate for the switching time is when the ABO is about to overcome the money in the DC account. (See the FRS website, for example.) But this is a complete optical illusion since it is growth and investment returns – not size – that matter. Instead, it depends on the 'gap' between the contribution rate b_s on the DB account, and the actual contribution c_s on the DC account. Intuitively, the value of b_s starts out quite low, and then overtakes c_s. Once the 'gap' is large enough, it is optimal to switch. Furthermore, when $\mu = \rho$, which means that the valuation rate is equal to the rate of return in the DC investment account, the optimal time to make the second election, is as soon as b_s overtakes c_s.

Interestingly, in the event that $\rho > \mu$, for example, when the valuation interest rate is unnaturally high and the ABO price is artificially low, the optimal time to make the second election is even earlier, which is when $b_{s^*} > c_{s^*}$.

One can re-write the critical first order condition, using equations (7) and (3), as:

$$\left(\frac{1}{s^* + \tau} + g\right) B_{s^*} - \widehat{c} I_0 e^{g s^*} = (\mu - \rho)B_{s^*}, \tag{12}$$

which, after dividing by B_{s^*}, and some algebra, leads us to:

$$\frac{1}{s^* + \tau} - \frac{\widehat{c}}{k(s^* + \tau)e^{s^* \rho}} = \mu - (\rho + g) \tag{13}$$

Finally, collecting terms and invoking the result mentioned above that for suitable μ the second derivative is negative at the first critical point of $W_T[s]$, the optimality condition can be stated as:

$$\phi(s) := \frac{1 - \frac{\widehat{c}}{k} e^{-\rho s}}{(s+\tau)} \geq \mu - (\rho + g), \tag{14}$$

where, we remind the reader once again, that $k = \widehat{b}a_x e^{-\rho T} = \widehat{b}a_x e^{-\rho(y-x)}$ which is the accrual-rate scaled present value of the DB pension annuity.

Equation (14) contains the main ideas of this section, and possibly the paper, so we note the following observations.

1. For younger employees the ratio $\widehat{c}/k > 1$. In other words, the DC contribution rate is much higher than the accrual-rate scaled present value of the DB pension annuity. Thus, $1 - \widehat{c}/k < 0$, and $\phi(0) < 0$. In this case, the global (qualitative) behavior of the function $\phi(s)$ is as follows: It starts in negative territory – since $\phi(0) < 0$. It then increases and crosses zero once, before reaching a maximum value in the positive quadrant. Then, it starts to decline and eventually converges to zero in the limit. Figure #1 (in the appendix) illustrates the behavior of this function.

2. If the accrual-rate scaled present value of the DB pension annuity k is large relative to the DC contribution rate $\widehat{c}$ – which would be applicable to older employees with more service credit – there is a possibility that $\phi(s) \geq \mu - (\rho + g)$, for all values of s, which would imply that $s^* = 0$. The optimal policy is an immediate switch to (or staying in) the DB plan. This, once again, should make sense given the fact that longer periods of service are associated with a higher ABO and thus a greater incentive to remain in the DB plan.

3. See Figure #1 and Figure #2 for a global and local picture of the behavior of $\phi(s)$ for three different examples. They are all based on an employee that has $\tau = 3$ years of service, who plans to retire at age $x = 60$. In the first example, the employee is $y = 45$ years-old, and is $T = 15$ years from retirement. In the second example, the employee is $y = 43$ years-old, and has $T = 18$ years to retirement, and in the third example the employee is $y = 40$ years-old and has $T = 20$ years to retirement. The valuation rate is $\rho = 8\%$, the investment (expected) return within the DC plan is $\mu = 12\%$, the salary

growth rate is $g = 4.75\%$, the DB rate is $\widehat{b} = 1.6\%$, and the DC contribution rate is $\widehat{c} = 9\%$ of salary.

4. If, at some point over its employment domain $s \in [0, T]$, the function $\phi(s)$ 'hits' the value of $\mu - (\rho + g)$, we have an interior solution to our first order condition, and that is the optimal time s^* to make the second election. This is consistent with our earlier observation in equation (11) that, in most cases, the b_s (contributions to the DB plan) start at a lower value than c_s (contributions to the DC plan). The optimal switch (second election) point occurs if and when $b_s - c_s \geq (\mu - \rho)B_s$.

5. We have completely moved away from expressing the optimal switching rule as a function of either the balance in the DC account C_s, or the actual value of the ABO, denoted by B_s. In fact, under our assumptions, the size or magnitude of the account is irrelevant to the decision. As we shall see clearly in the numerical examples, the optimal switching time s^*, occurs *prior* to the time when $C_s = B_s$. And, in some cases, by quite a number of years. Intuitively, the participant makes the second election – back into the DB plan – and keeps the difference $C_{s^*} - B_{s^*}$ invested and growing in the DC plan.

6. All else being equal, the accrual rate scaled present value of the DB pension annuity $k = \widehat{b} a_x e^{-\rho T}$ will increase, the closer the participant is to retirement, since the implied value of T will be lower. Thus, a shorter time to retirement will have the same impact at a longer service record τ. They both act to increase that value of the function $\phi(s)$, and thus accelerate the point at which the second election should be made.

7. It is relatively easy to prove that $\phi(s) + \rho$ is an increasing function of the valuation interest rate ρ. Thus, the function $\phi(s)$ will shift closer to $\mu - (\rho + g)$, as ρ increases, and thus the switching time will be earlier. This, once again, should be consistent with intuition. As the valuation interest rate increases, there is a greater incentive to be in the DB plan.

8. Finally, note from equation (14) that, as long as $\phi(s) < \mu - (\rho + g)$, the participant is *better off* staying in the DC plan. Thus, one can invert equation (14), and express the entire decision as a function of μ, as opposed to a function of s. Stated differently, we

> can then locate the *required rate of return* needed to justify staying in the DC plan. This can be formally represented as:
>
> $$\mu > \phi(s) + (\rho + g) \tag{15}$$
>
> As long as the value of μ is large enough, one should stay in the DC plan, but, as soon as μ *slips* under $\phi(s) + (\rho + g)$, it is optimal to switch. Recall that $\phi(s)$ starts at a negative value, and then moves into the positive region. At some point, $\phi(s) + (\rho + g)$ might exceed μ. If this occurs within the range $s \in [0, T]$, the second election is made. If, this occurs, mathematically, outside the range $s \in [0, T]$, or if this never occurs, the participant should stay in the DC plan until retirement.

As a prelude to the stochastic model – in which the investment return in the DC account is uncertain – one can envision giving advice in the following manner. At each point in time, the participant in the DC plan is given a threshold investment return μ that is required to justify staying in the DC plan. For example, he or she might be told that "you must earn at least 6% in your DC account to justify staying in the DC plan for the next month". Table #1 (in the appendix) provides some numerical estimates of this 'threshold return' under a variety of entry ages and initial service credit years.

In the same manner, the participant could be given a statistically (simulated) distribution for the (instantaneous) return in their DC account, from which they can observe the probability of earning the threshold return. Thus, on a more sophisticated level, one might be informed that "There is a 72% chance your portfolio will earn enough to justify staying in the DC plan".

2.3 The AAL and New Employees

While most of the above analysis centres around employees currently in the Florida retirement system – for whom the price of returning is based on the ABO - a similar analysis can be conducted for new employees and the accumulated actuarial liability (AAL). Without getting into the technical details, we can model the evolution of the *price of re-entry* into the DB plan, by modeling the (new) B_s as an integral of the (new) b_s. Obviously, some of the subjectivity involved in the actuarial method of funding would reduce the precision by which we could project B_s. In any event, we would then locate the optimal time s, that maximizes equation (1), using the new definition of B_s and b_s. In fact, this would lead to a first order condition that is identical to equation (11). Our numerical results indicate the given the high cost of the AAL, for somebody in the DC plan, the optimal time to make the second election – if at all – is right before retirement. We deliberately refrain from pursuing this line of research since our objective is to analyze the value for the current 600,000 employees, and because the AAL is highly dependent on (subjective) actuarial assumptions.

2.4 Numerical Examples.

We now present some actual numbers to get a practical sense of the optimal timing and incremental value of Florida's *2nd election.* Our parameters are mostly based on actual Florida State actuarial assumptions.

2.4.1 Case #1.

We start with a $y = 40$ year-old employee with $\tau = 7$ years of service, currently earning $I_0 = \$30,000$, which is expected to grow by $g = 4.75\%$ per annum. The employee accrues $\widehat{b} = 1.6\%$ of salary for each year of service and plans to retire at age $x = 60$ which is in $T = 20$ years. In a $\rho = 8\%$ valuation environment, where the annuity factor is $a_{60} = 14.75$, one obtains a current ABO of $B_0 = \$10,005$.

This employee rationally makes the initial election to switch into the DC plan and rightfully transfers the ABO of $\$10,005$ into the DC plan, so that $C_0 = \$10,005$, as per the discussion in the earlier section. Accordingly, if we assume a $\widehat{c} = 9\%$ of pay contribution rate and a *subjective* assumed investment return of $\mu = 8\%$ (same as the valuation rate) in the DC account, the retirement value of the DC account will be $C_{20} = \$246,225$, assuming a second election is not made. If, however, an employee were to remain in the DB plan, the retirement value of the ABO would be $\$494,284$, as we calculated earlier.

Thus, without the availability of the second election, it would be suboptimal – under these parameter values – to ever elect the DC plan, since $\$246,225$ is clearly less than $\$494,284$. However, with ability to make a second election, it initially becomes optimal to move into the DC plan, and then return at time $s^* = 3.06$ years. This number comes directly from equation

(14). The participant will then rightfully spend the last 17 years of his or her working life in the DB plan. Note that at time $s^* = 3.06$, the value of the ABO will be $B_{3.06} = \$21,242$, while the money in the DC account will have grown to $C_{3.06} = \$22,827$. The remaining \$ 1,585 in the DC account upon the second election, will grow during the remaining 17 years, to \$6,146. The retirement wealth will consist of the DB account \$494,284, plus the left over money in the DC plan; all of which accumulates to $W_{20}[3.06] = \$500,430$.

Note once again that the optimal time to switch is not when the value of the ABO is about to exceed the value of the DC account. In fact, that would not occur until $s = 6$ years. Indeed, if the participant were to wait until $s = 6$ to make the second election, he or she would be no better off than staying in the DB plan to begin with since $B_6 = C_6 = \$40,000$. Indeed, the value of the 2nd election comes from being able to buy into the ABO, *and* keep some leftovers. The greater the leftovers and the longer the period over which it accumulates, the higher the value of the 2nd election.

To be more precise and consistent with our definition in equation (2), the incremental value of having the *2nd election* is

$$v_{20}[3.06] = \frac{\$500,430}{\$494,284} - 1 = 1.24\%. \tag{16}$$

This captures the relative gain from being able to move into, and then out of, the DC plan. Notice that the 2nd election value is relatively small at 1.24% of final DB retirement wealth.

We emphasize once again that, were it not for the ability to make the second election, it would be suboptimal for this employee to participate in the DC plan to begin with. The reason is that with $\tau = 7$ years of service, and a relatively low $\mu = 8\%$ (subjective estimate for the) rate of return in the DC plan, there is no reason to leave the DB plan.

2.4.2 Case #2.

We continue with the same employee set-up, except that in this case the employees subjectively assumes the DC account balance will grow at $\mu = 12\%$, instead of 8%. Thus, while the retirement value of the ABO remains at $B_{20} = \$494,284$, the retirement value of the DC account, assuming the initial ABO of \$10,006 – and all future contribution – are invested in the plan, is $C_{20} = \$424,520$. Once again, in the absence of the second election, it would be suboptimal to ever move into the DC plan. Once the second election is granted, however, the optimal time to switch back is (later) in $s^* = 6.725$ years. In this case, retirement wealth will be $W_{20}[6.725] = \$535,542$, and the incremental value from the *2nd election* is a more substantial:

$$v_{20}[6.725] = \frac{535542}{494284} - 1 = 8.3\% \tag{17}$$

Upon the second election, the value of the DC account is $C_{6.7} = 54,630$, while the value of the ABO in the DB account is $B_{6.7} = 46,244$.

2.4.3 Case #3.

We assume an employee age $y = 36$, with $\tau = 1$ year of service and an expected retirement age of $x = 60$, or in $T = 24$ years. The employee is currently earning $I_0 = \$30,000$, which is expected to grow by $g = 4.75\%$ per annum. The employee accrues $\widehat{b} = 1.6\%$ of salary for each year of service. In a $\rho = 8\%$ valuation environment, when the annuity factor $a_{60} = 14.75$, one obtains a current ABO of $B_0 = \$1,038$. If this employee stays in the DB plan until retirement, his or her ABO will be $B_{24} = \$553,438$. If this employee decides to participate in the PEORP and move his ABO to the DC plan, his retirement wealth will be $C_{24} = \$565,474$, in a $\mu = 12\%$ environment.

This employee should therefore switch into the DC plan, and then return at $s^* = 10.88$. His total retirement wealth will be $W_{24}[10.88] = \$695,555$, which leads to a value of:

$$v_{24}[10.88] = \frac{\$695,555}{\$565,474} - 1 = 23\% \tag{18}$$

One way to put this value in context, is that to create the same level of wealth at retirement – without the buyback ability – the employer would have to increase the contribution rate by 23%, i.e. from 9% of current salary to 11.1% of current salary.

2.4.4 The Hurdle Rate approach for the Optimal Switching Time

Consistent with the earlier idea inverting our optimality condition and expressing the optimal (second election) switching time as a function of μ, we offer the following Table #1. The assumptions are identical to the earlier cases. The employee earns $\widehat{b} = 1.6\%$ of salary for each year of service. In a $\rho = 8\%$ valuation environment, when the annuity factor $a_{60} = 14.75$. The DC contribution rate is $\widehat{c} = 9\%$, and the retirement age is $x = 60$.

For example, a 35 year-old employee that joined the Defined Contribution plan at age 30 – who had 5 years of service at that time – would have to earn at least 6.8% to justify staying in the DC plan. However, once this same employee reaches age 50, the required threshold increases to 13.3%, which is more difficult to achieve and will have a higher probability of failure. Notice that at young enough ages, the threshold investment return is *negative!* This is because the 9% contributions of the employer to the DC plan are so high, relative to the contributions to the DB plan, that a participant would be able to lose money in the DC account, and still come out ahead.

We must reiterate once gain that, in the simplified model of this section, we are not taking into account the extra risk that may be incurred in the DC portfolio. As a practical matter the employee would have to consider any additional risk taken to earn the higher threshold investment return, in addition to their capacity for borrowing outside of the DC plan. Thus, for example, *even* if the employee were confident that they could earn the threshold return described above, it is by no means clear that they should continue in the DC plan. They may be better off switching into the DB plan and transferring other investments or borrowed funds into the riskier assets with the higher (subjective) expected returns. And, for an employee who is liquidity constrained and can not borrow, they too might be better off switching to avoid the excessive risk which is present in the DC plan.

Indeed, all of these factors are taken into account in Section 3 where we discuss the more realistic stochastic approach, which leads to a risk-neutral valuation.

2.4.5 The Impact of μ on the Incremental Value of the 2nd Election

Interestingly, when we raise the investment return in Case #3, to $\mu = 14\%$, we obtain that although $C_{24} = \$778,948$, $s^* = 14.67$ years, and $W_{24}[14.67] = \$817,514$, the actual value, or increase in final wealth from having access to the second election, is only $v[14.67] = 4.9\%$, which is much lower than for the case $\mu = 12\%$. At first it might seem puzzling that, if the return from the DC account is greater, then how the can the incremental value of the 2nd election to switch back be lower? However, this apparent paradox is resolved by realizing that although the final value of W_T is much higher, compared to when $\mu = 12\%$, the incentive to *ever leave* the DC plan is reduced as well. In fact, when $\mu = 16\%$, which is unrealistically high, of course, it is never optimal to make the second election. Rather, the employee stays in the DC plan until retirement. This is a direct result of equation (14). When, $\mu - (\rho + g)$ is large enough, the function $\phi(s)$ will never reach $\mu - (\rho + g)$ in the time interval $s \in [0, T]$.

Indeed, on a global level, there is a relatively narrow range of returns for which the second election has value. If μ is large enough, the participant switches to the DC plan, and remains there until retirement. In Case #3, the threshold μ for never switching back would be 15.3%.

In the extreme case, an employee with $\tau = 1$ years of service, who is currently aged $y = 31$ and who expects to retire at age $x = 60$, would make the second election at time $s^* = 13.43$ (which is age 43) and would derive a value of $v[13.43] = 30.8\%$. In other words, his final retirement wealth would increase 30.8% by electing to switch at the optimal time. Once again, this translates into an equivalent DC contribution rate of $\widehat{c} = 10.75\%$.

In sum, we are hard pressed to find realistic cases in which the incremental value is worth more than 30%, or a case for which the equivalent defined contribution rate is much higher than $\widehat{c} = 11\%$.

2.4.6 From the State's Point of View

We now shift perspectives and examine the problem from the point of view of the State. Even if we can not predict exactly when employees will switch from the DB plan to the DC plan and back – since this depends on rationality, personal attitudes towards risk as well as preferences for annuity payments over cash – we can provide a financial *upper bound* on the cost of providing the 2nd election. The upper bound comes from assuming the employee behaves in a game theoretic manner that maximizes the cost of the 2nd election to the State. We now show how this upper bound actually collapses to the above-mentioned deterministic case so that our previous discussion and estimates are equally valid.

Following our earlier notation for c_t and b_t, we introduce $V_T[s]$, which captures the financial cost to the State of providing the DB and DC pension *together* with the 2nd election. Note that

$$V_T[s] = \int_0^s c_t e^{\rho(T-t)} dt + \int_s^T b_t e^{\rho(T-t)} dt, \tag{19}$$

which is the future value of the State's total DC and then DB pension contributions, assuming the employee switches from DC back to DB at time s. The future value is computed based on the valuation rate ρ. Note from equation (4) and (8) that the cost to the State can be expressed as:

$$V_T[s] = C_s e^{\rho(T-s)} + B_T - B_s e^{\rho(T-s)} = B_T + (C_s - B_s)e^{\rho(T-s)}, \tag{20}$$

which is precisely equal to equation (1), but with μ replaced by ρ.

In other words, the switching time that maximizes the cost to the State is precisely the switching time that maximizes the benefit to the employee, provided the DC account is assumed to earn $\mu = \rho$, which is the valuation rate. Therefore, according to equation (11) the switching time s that maximizes the cost to the State is precisely when $b_s = c_s$. Furthermore, given the concavity of the function $V_T[s]$ with respect to s, *if the employee switches at any other time the cost to the State will be lower.* This point is critical as we move forward to a stochastic model in which rates of return from the individual DC account are uncertain.

3 Stochastic Case: Dealing with Uncertainty.

We want to abstract from optimal asset allocation questions by assuming that the employee has a pre-existing portfolio outside the pension system. This portfolio will be used to supplement the purchase of the ABO benefit (if needed) and will also be used to fine-tune the asset mix between risky and risk-free investments. In other words, we assume the participant is not liquidity constrained and can therefore invest in (and sell from) the same assets that are available in the DC plan. If this is not the case, the optimal switching time will differ from the one derived bellow, but the cost to the State will actually be lower as per the previous discussion.

Consistent with the continuous-time asset pricing literature – as originally employed by Black and Scholes (1973) – we will assume that the instantaneous rate of return in the DC account is no longer μ, but is replaced with a random variable $\widetilde{X}_t$.

The stochastic process $\widetilde{X}_t$ is modeled as a non-standard Brownian motion under the P-measure, with instantaneous drift $\mu - 0.5\sigma^2$ and volatility, or standard deviation σ. Algebraically:

$$\widetilde{X}_t = (\mu - 0.5\sigma^2)t + \sigma B_t, \tag{21}$$

where B_t is a standard Brownian motion and so the process $e^{\widetilde{X}_t}$ is a Geometric (exponential) Brownian Motion. The drift term in equation (21) is consistent with techniques from stochastic calculus, so that we end-up with a P-measure expectation of:

$$E_P[e^{(\widetilde{X}_T - \widetilde{X}_s)}] = e^{\mu(T-s)}. \tag{22}$$

We use the symbol $E_P[.]$ to denote the physical (or statistical) expectation of the process under a real-world measure. The risk-adjusted (a.k.a risk-neutral) expectation under the Q-measure, which we will soon invoke, will be denoted by the symbol $E_Q[.]$.

As before, the stochastic process for the DC account can be modeled by:

$$\widetilde{C}_s = \widetilde{C}_0 e^{\widetilde{X}_s} + \int_0^s \widehat{c}_t e^{(\widetilde{X}_s - \widetilde{X}_t)} dt, \tag{23}$$

and the value of the DB account will be:

$$\widetilde{B}_s = \widetilde{B}_0 e^{\rho s} + \int_0^s \widehat{b}_t e^{\rho(s-t)} dt. \tag{24}$$

If we further make the assumption that valuation rates (ρ), salary growth rates (g) and accrual rates ($\widehat{b}$) are constant, the expected value of the Defined Benefit ABO is the same as in the deterministic case, which was presented in equation (6).

It is important to emphasize that the valuation rate ρ is quite distinct from (and consistently higher than) the risk-free rate. Thus, part of the value from participating in the DB plan is that benefits grow at an artificially higher rate. As we illustrated earlier, the greater the valuation rate ρ, the greater is the incentive to move (back) into the DB plan.

We are now faced with a question of what the employee seeks to maximize in a random environment. Since wealth is random at retirement – and is dependent on the performance of the underlying DC account – one might argue that employees should be maximizing the utility of terminal wealth. And indeed, most employees will probably make such an evaluation. However, assuming the employee has access to the same financial instruments both inside and outside the DC plan, the actual riskiness of the DC return should not influence the optimal switching time. Rather, the employee should treat the optimal stopping problem in a complete market setting in which the relevant expectation is the risk-adjusted Q-measure.

Thus, the expected wealth at retirement – under the risk adjusted Q-measure – assuming the 2nd election is made at time s, is:

$$\begin{aligned}
E_Q\left[\widetilde{W_T}[s]\right] &= E_Q\left[\widetilde{B}_T + (\widetilde{C}_s - \widetilde{B}_s)e^{(\widetilde{X}_T - \widetilde{X}_s)}\right] \\
&= \widetilde{B}_T + \left(E_Q\left[\widetilde{C}_s e^{(\widetilde{X}_T - \widetilde{X}_s)}\right] - E_Q\left[\widetilde{B}_s e^{(\widetilde{X}_T - \widetilde{X}_s)}\right]\right) \\
&= \widetilde{B}_T + \left(E_Q\left[\widetilde{C}_s e^{(\widetilde{X}_T - \widetilde{X}_s)}\right] - E_Q\left[\widetilde{B}_s\right] E_Q\left[e^{(\widetilde{X}_T - \widetilde{X}_s)}\right]\right) \\
&= \widetilde{B}_T - \widetilde{B}_s e^{r(T-s)} + \left(E_Q\left[\widetilde{C}_s e^{(\widetilde{X}_T - \widetilde{X}_s)}\right]\right). \qquad (25)
\end{aligned}$$

Now, in order to maximize the (risk neutral) expected wealth at retirement we seek to maximize the quantity

$$E_Q[\widetilde{C}_s e^{(\widetilde{X}_T - \widetilde{X}_s)}] - \widetilde{B}_s e^{r(T-s)}, \qquad (26)$$

over all times s. Some readers may initially believe that *ex ante,* the switching time should depend on the amount of money within the DC plan and therefore, the maximum should be taken over all possible *stopping times* rather than over all definite times s. And, while this type of approach is required for various American option pricing problems, it is not needed in our case. We will elaborate on this in a moment, but for now refer the interested reader to the appendix for a formal proof of this fact.

Note once again that in the event the investor is liquidity constrained and can not borrow money outside the DC plan in order to invest in the risky asset, the optimal switching time will satisfy:

$$\max_s E_P\left[U(\widetilde{W_T}[s] + \mathbf{Z})\right], \qquad (27)$$

where $U(.)$ represents the employee's utility function of wealth, the expectation is based on the physical measure $E_P[.]$ and $\mathbf{Z}$ denotes the employee's background wealth which will influence the optimal switching time as well as external asset allocation and leverage decisions. Solving equation (27) is beyond the scope of this paper – and we leave this for future research – since we are currently concerned with the additional economic cost of providing the 2nd election, as opposed to the investor's unique (utility maximizing) behavior.

Combining equation (25) and equation (23), we obtain:

$$\begin{aligned}
E_Q\left[\widetilde{C}_s e^{(\widetilde{X}_T - \widetilde{X}_s)}\right] &= E_Q\left[\int_0^s \widehat{c} I_0 e^{gt} e^{(\widetilde{X}_T - \widetilde{X}_t)} dt\right] \\
&= \int_0^s \widehat{c} I_0 e^{gt} E\left[e^{(\widetilde{X}_T - \widetilde{X}_t)}\right] dt = \int_0^s \widehat{c} I_0 e^{gt} e^{r(T-t)} dt \\
&= e^{rT}\int_0^s \widehat{c} I_0 e^{(g-r)t} dt = \frac{\widehat{c} I_0 e^{rT}}{g-r}\left(e^{(g-r)s} - 1\right) \qquad (28)
\end{aligned}$$

The second equality, which interchanges the expectation and integral, is allowed by Fubini's Theorem. In essence, we are computing the risk-neutral expectation of an Asian option with a zero strike price, which is an Asian forward contract. See Milevsky and Posner (1998) for more information on Asian options and the techniques for computing the relevant expectations. This leaves us with a value for $E_Q[\widetilde{W_T}[s]]$ that is identical to the deterministic situation in equation (1) and (9).

Formally, the optimal 2nd election switching time (a.k.a. stopping time) is the first time that s satisfies:

$$\phi(s) := \frac{1 - \frac{\widehat{c}}{k} e^{-\rho s}}{(s + \tau)} \geq r - (\rho + g). \tag{29}$$

The independence of the decision on the actual performance (and value) of the DC plan, including the randomness – in contrast to the classical American option pricing problem – can also be explained intuitively. See we are assuming that the individual has access to other sources of capital either by borrowing, or via assets outside the DC fund, we may view the switching decision as follows. We can in effect assume that the person is always going to keep their DC plan intact up to time T. The decision is therefore *not* when to switch from DC to DB, but when to *buy into* the DB plan. The cost of doing so is as follows. To start, the individual must make the payment of B_s at time s. In addition, they must, in effect, make payments at the rate of c_l at time l for $s \leq l \leq T$. These are the contributions to the DC plan which they are no longer receiving. In other words, the problem is reduced to simply finding the time s^* which minimizes the value of a payment B_s at time s, plus the payments at a rate of c_l after time s. Clearly, these payments are independent of the accumulated wealth in the DC plan.

Those familiar with the financial option pricing literature might wonder where the volatility σ – which was part of the dynamics of the risky return $\widetilde{X}_t$ – is taken into account in the optimal policy. Here lies the crux of our non-option argument. We are not really faced with a traditional non-linear payoff structure. Rather, our payoff profile is linear in the underlying stochastic variable. If the analogy to a derivative contract is to be used, then the 2nd election is akin to a forward contract, and the option is invariant to return volatility. The intuition for this follows directly from the strike price of this so-called option. The cost of making the 2nd election – which is the ABO – is the same regardless of the performance of the underlying DC account value. Thus, the participant is not shielded from downside risk and participates dollar for dollar in any market declines.

In summary, the optimal time to make the 2nd election in the presence of DC return uncertainty is when the left-hand side of equation (29) equals or exceeds the quantity $r-\rho-g$. The economic value of the 2nd election is defined equal to the proportional increase in retirement wealth that would be applicable in the absence of the 2nd election.

3.0.7 A Precise Value Using Employee Data.

Table #2 and Table #3 (in the appendix) provide a snapshot of the Florida retirement system in late 2001. The matrix displays the number of employees in various 'age bucket' as a function of their years of service. For example, 2.75% of the labor force is less than 22 years old and have less than 2 years of service. In contrast, 1.78% are between the ages of 22 and 27, and have between 2 and 7 years of service, etc. In the upper corner we observe that only 0.02% of the Florida public labor force are between the ages of 57 and 62 and have between 37 and 42 years of service. Table #3 displays the average (current) salaries in each of these 5-year buckets. Quite intuitively, the older employees (with more service credit years) have higher salaries.

Finally, Table #4 provides 2nd election values – as per equation (2) – for the employees at the upper corner of the various buckets. Once again, these values represent the increase in equivalent retirement wealth that is attributable to the 2nd election. Thus, for example, an employee that is 22 years of age, and that has 2 years or credited service, would gain 23.4% from having the 2nd election. In contrast, employees that are 47 years of age, or older, would assign zero value to the 2nd election since they would be better off staying in the DB plan, or never electing the DC plan to begin with. Once again we stress that a zero value does not imply that it is irrational to elect the DC plan at advanced ages. If the participant decides to stay in the DC plan or defer the 2nd election, the State will save money.

3.0.8 Comparison with Lachance, Mitchell and Smetters (2003)

In a related paper that appeared in the *Journal of Risk and Insurance*, Lachance, Mitchell and Smetters (2002) analyzed the same Florida 2nd election using Monte Carlo Simulation (MCS) techniques. They pointed out similarities with the classical Magrabe (1978) 'option to exchange' and derived a variety of employer cost estimates using a range of ages and employment termination probabilities. They concluded that "the market value of this option could represent up to 100 percent of the DC contributions over the worklife".

While the current paper differs in methodology, we are hard pressed to find cases in which the value of this so-called option exceeds 30% of the DC contribution rate. Furthermore, the value of the 2nd election is actually the highest for younger employees, but they are the ones that are most likely to terminate employment prior to retirement. Thus, from an actuarial funding perspective, the cost of providing this 2nd election to a large group of young employees – for which the law of large numbers would apply – would be much smaller than 30%.

We believe the LMS (2002) study overestimates the incremental liability created by the 2nd election. We believe the crux of the problem can be traced to their equation (4) on page 5 (Volume 70, Number 1). That equation contains the sum of two terms for the expected cost of the buy-back option at the inception of the DC plan. Their first term in that equation is meant to capture the State's gain/loss due to the difference between the DB and DC benefits, and the second term captures the difference between the State's past contributions and the DB buy-back price. However, their first term should not be part of the equation since this cost would been incurred *even* in the absence of the 2nd election. The additional cost of the 2nd election should only consider cash-flows that would have not existed otherwise. Indeed, the State would have been obligated to pay the same DB pension had the employee never been granted the 2nd election.

Furthermore, while the LMS (2003) paper randomized the risk free rate, the salary growth rate, the investment returns in the DC and DB plan as well as the contribution rate to the DB plan, our analytic model can be used on any given (extreme) range of parameter value. *Finally, as we argued above, the linearity of the payoff structure implies that uncertainty in the stochastic variables will not add to the cost of providing this guarantee above and beyond the normal DB funding costs.*

We emphasize that the 2nd election is best described as a forward contract that enables the employee to acquire a DB pension at a reduced price – similar to an in-the-money forward contract – as opposed to an option with downside protection. In contrast to standard put option pricing analogy, the employee has absolutely no incentive to increase the volatility of the investment returns in the DC account since they will have to make-up any difference in the amount needed to buy-back with their own personal (tax sheltered) funds.

3.0.9 What Buy-Back Price Will Create Zero Incremental Value?

Instead of charging the ABO to an employee wishing to switch from the DC plan to the DB plan, there is an alternative scheme which would effectively reduce the incremental value of the 2nd election to zero. This alternative is to demand a payment of (or set the strike price to be) all the contributions that had been made to the DC plan, accumulated with interest, *at the valuation rate* ρ. In other words, the employer would restore the employee to the DB plan, assuming they return all the funds received, but accumulated at the employer's valuation rate.

Of course, if the employee managed to earn a rate of return that is higher than the valuation rate in the DB plan, the employee will retain the excess earnings. The possibility of extra earnings was, after all, the incentive to participate in the DC plan in the first place. On the other hand, if the employee has earned less than the valuation rate, they will have to pay back more than what they have in their fund, which should make intuitive sense.

Looking at the mathematics of the situation, the cost to the employee wishing to switch from DC to DB at time s would be:

$$B_s = B_0 e^{\rho s} + \int_0^s c_t e^{\rho(s-t)} dt$$

Note that this is the same as B_s defined in equation (8) except with c_t replacing b_t. Following the derivation as above leads to a revised equation (11) where the left hand side is zero. We therefore see, as was intuitively clear, that when $\mu = r$, the equation is satisfied by all s^*. In other words, the final wealth at retirement is constant regardless of the time of switching. This statement is echoed in LMS (2003) as well.

4 Conclusion

This paper has analyzed an intriguing pension choice that has been offered to all 600,000 employees in the State of Florida's newly privatized pension system. The so-called buy-back option allows the employee to initially switch from a Defined Benefit pension plan to a Defined Contribution plan, and then back again at any point prior to retirement, by paying the accumulated benefit obligation (ABO). The ABO is essentially the present value of their pension benefit based on years of credited service.

The Florida pension reforms are being viewed by many observers as a precursor to Social Security reform in U.S., which elevates this problem to more than just a quirk in a particular pension plan.

Our simple, yet robust model provides us with the following economic insights.

- The optimal time to make the 2nd election is a function of the *rate of change* in the accumulated benefit obligation – compared to the growth rate of the DC account – and is not a function of the size of the account *per se*.

- We concur with Lachance, Mitchell and Smetters (2003) that the 2nd election is valuable – especially to young employees – and is likely to be exercised by most participants who initially elect the DC plan. In fact, in our basic model which (deliberately) ignores mortality and early termination probabilities, *nobody* retires from the DC plan; rather, they all eventually return to the DB plan.

- While the educational material developed by the State and Lachance, Mitchell and Smetters (2003) consider the 2nd election as an option, in truth, the liability created by this provision behaves more like a (linear) forward contract as opposed to a (non-linear) put option. Thus, for example, increasing the volatility of the underlying securities within the DC plan will not increase the value – because the participants stand to lose as much as they stand to gain – which is in contrast to traditional option pricing models.

- We are hard pressed to find realistic combinations of parameter values that induce a 2nd election value that is greater than 30%. When translated into Defined Contribution terms, one would have to raise the contribution rate from 9% to at the absolute most 12% – for those who elect the DC plan – in order to create a final wealth profile that is equivalent to the absence of the 2nd election. Our results differ from the analysis of Lachance, Mitchell and Smetters (2003), who we believe overestimate the value of this so-called option by not separating it from the pre-existing DB pension liability of the State.

- When one factors in the probabilities associated with reaching the optimal switching time – and the rationality involved in making this decision – we believe that the actuarial funding cost of the buy-back option is much lower than 30% of the DC plan contribution rate. Indeed, according to State of Florida estimates, a 34 year-old male with 2 years of service has less than a 25% probability of making it to normal retirement age.

Ongoing research is examining the impact of randomizing the quantities that we held fixed thru-out the paper, such as interest rates r, salary growth rates g, discount rates ρ, and retirement dates T on the optimal *utility maximizing policy of the employee.* However, despite the substantial (and enjoyable) intellectual capital that is spent in this endeavour, we find that greater uncertainty does little to add value to the 'second election', which we believe is mainly due to the linearity of the payoff profile.

Finally, given the abundance of empirical evidence – see for example Benartzi and Thaler (2001, 2002) – that many employees are not making fully rational and utility maximizing choices within their pension plan, it will be fascinating to observe what fraction of Floridians will behave rationally with their 2nd election. Indeed, according to recent estimates by the State, less than 10% of the in-force employees elected to convert their DB into a DC plan.

5 Appendix:

5.1 Appendix 1

In this appendix we provide the (deterministic) conditions under which the value of wealth at retirement in maximized by the s^* defined by

$$b_{s^*} - c_{s^*} = (\mu - \rho) B_{s^*}, \tag{30}$$

which is the first order condition listed in the body of the paper. Specifically, we now demonstrate the for a suitably restricted range of values for μ and the ratio $\widehat{c}/k$, the point s^* will in fact result in a global maximum for W_T. First, let us define the quantity $\delta := \mu - \rho - g$, which is the expected return from the DC individual account net of the valuation rate and the growth rate of salary. We consider the function $W_T[s]$ defined on the interval $[0, T]$.

Theorem.

There are either zero, one or two solutions in the first order condition given in equation (30).

(a) Suppose that $\widehat{c}/k > 1 - \delta\tau$. If there are no solutions to equation (30), the maximum of $W_T[s]$ will occur at $s = T$. This will always be the case if $\delta \geq \rho$. In the case that one or two solutions exist the following holds. Let s^* denotes the smallest solution.

(i) If $\delta \leq \rho/((1+\rho(T+\tau))$ the maximum of $W_T[s]$ occurs at s^*. (ii) If $\rho/((1+\rho(T+\tau)) < \delta < \rho$ then $W_T[s]$ will either have an inflection point at s^*, in which case the maximum will occur at $s = T$, or a local maximum at s^* in which case the maximum will occur at either $s = s^*$ or at $s = T$.

(b) Suppose that $\widehat{c}/k \leq 1 - \delta\tau$. The maximum of $W_T[s]$ will occur at either $s = 0$ or $s = T$. The former always holds if $\delta \leq \rho/((1 + \rho(T + \tau))$.

Proof.

Going back to the definition of B_s and C_s and then taking derivatives leads to:

$$W_T'[s] = K(s)\left[\frac{\widehat{c}}{k}e^{-\rho s} - (1 - (s+\tau)\delta)\right], \tag{31}$$

for some positive quantity $K(s)$ and

$$W_T''[s] = I_0 k\left[\frac{\widehat{c}}{k}(-(\rho+\delta))e^{-(\rho+\delta)s} - (\delta^2(s+\tau) - 2\delta)e^{-\delta s}\right]. \tag{32}$$

>From equation (31) solutions of equation (30) can only occur for those points s where the straight line $1 - \delta(s+\tau)$ meets the convex curve $(\widehat{c}/k)e^{-\rho s}$. This can occur in at most two places if the line begins above the curve at $s = 0$ or in at most one if the line begins below the curve. At any such intersection point s we can substitute into equation (32) to get (assuming that $\delta \neq 0$) that:

$$W_T''[s] = I_0 k e^{-\delta s}(-\delta)\left[\rho\left(\frac{1}{\delta} - (s+\tau)\right) - 1\right]. \tag{33}$$

(a) In this case, the line begins below the curve at $s = 0$ so that $W_T'[s]$ starts off positive. Suppose that a second intersection point s^{**} exists. (In the case that the line is tangent to the curve we consider this as a double point and take $s^{**} = s^*$.) It is clear that $W_T[s]$ has an inflection point at s^{**} in the case of tangency, or a local minimum if not. This completes the proof of (ii). To show (i) we note first that since $W_T''[s^{**}] \geq 0$ we necessarily have $\delta > 0$

from equation (32), implying that the term in square brackets in equation (33) is less than or equal to 0 as s^{**}, which means that

$$s^{**} \geq \frac{1}{\delta} - \frac{1}{\rho} - \tau. \tag{34}$$

The upper bound on δ in (i) however shows that the right hand side of equation (34) is greater than or equal to T so that in this case the second intersection point cannot occur in the interval $[0, T]$. Finally, we demonstrate the statement at the beginning of (a). If $\delta > \rho$, then for $s > 0$

$$(\widehat{c}/k)e^{-\rho s} > (1 - \delta\tau)(1 - \rho s) \geq (1 - \delta\tau)(1 - \delta s) > 1 - \delta(s + \tau)$$

and there is no solution to equation (30).

(b) If $(\widehat{c}/k) < (1 - \tau)$ the line lies above the curve and there is at most one intersection point s^*. The derivative starts out negative and $W_T[s]$ will have a local minimum at s^* if it exists. When δ has the given upper bound we argue that as it part (a) to rule out occurrence of this point in the interval $[0, T]$ so that $W_T[s]$ is decreasing. With this bound, the derivative is also necessarily negative in the case that $\widehat{c}/k = (1 - \tau\delta)$ since the slope of the line $= -\delta > -\rho$, the slope of the curve as $s = 0$ and the curve will start below the line. **Q.E.D.**

The above theorem shows that the rule given by

$$\phi(s) := \frac{1 - \frac{\widehat{c}}{k}e^{-\rho s}}{(s + \tau)} \geq \mu - (\rho + g) \tag{35}$$

for the switching time can only lead us astray in the case of a(ii) of the theorem when $(\widehat{c}/k) > 1 - \delta\tau$ and $\rho/(1 + \rho(1 + T)) < \delta < \rho$, where we may possibly identify a local maximum which is less than the value of the function at $s = T$. The rule as stated will work vacuously in the case where $W_T[s]$ is monotone since the left hand side of equation (35) will either be always less than the right hand side or never less than the right hand side.

For a numerical example of the problem that might arise, suppose that $(\widehat{c}/k) = 0.75e^{0.4}$ and that $\rho = 0.8$, $g = 0.0475, \mu = 0.1775$, (granted, an unreasonably high value) $\tau = 0$ and $T = 20$. Then, $W_{20}[s]$ has a local maximum at $s = 5$. However, the individual should *not* switch at this point but instead stay in the DC fund. The resulting wealth at time $s = 20$ is almost 2.5 times as much as from switching at time $s = 5$.

5.2 Appendix 2:

To understand why we do not require a formal stopping time for the optimal switching time s^*, note that for any s,

$$\widetilde{C}_T = \widetilde{C}_s e^{(\widetilde{X}_T - \widetilde{X}_s)} + \int_s^T \widehat{c}_l e^{(\widetilde{X}_T - \widetilde{X}_l)} dl. \tag{36}$$

Consider two times $s_1 < s_2$ and let ξ denote the random variable $(W_T[s_1] - W_T[s_2])$. We will show that ξ is independent of $\widetilde{C}_{s_1}$. From equation (24) we have that:

$$W_T[s] = \widetilde{C}_T - \int_s^T \widehat{c}_l e^{\left(\widetilde{X}_T - \widetilde{X}_l\right)} dl - \widetilde{B}_s e^{\left(\widetilde{X}_T - \widetilde{X}_s\right)} + \widetilde{B}_T, \tag{37}$$

and so for any arbitrary value of z and s

$$\begin{aligned} & E_Q \left[W_T[s] \mid \widetilde{C}_{s_1} = z \right] \\ & = E_Q[\widetilde{C}_T \mid \widetilde{C}_{s_1} = z] - E_Q \left[\int_s^T \widehat{c}_s e^{(X_T - X_l)} dl \right] - E_Q \left[\widetilde{B}_s e^{\left(\widetilde{X}_T - \widetilde{X}_s\right)} \right] + \widetilde{B}_T, \end{aligned} \tag{38}$$

since the last two random variable are independent of the value of $\widetilde{C}_{s_1}$. Our claim follows immediately upon substituting s_1 and s_2 in the above and then subtracting. This shows that the decision to switch can not be altered by the particular value of the fund at that time.

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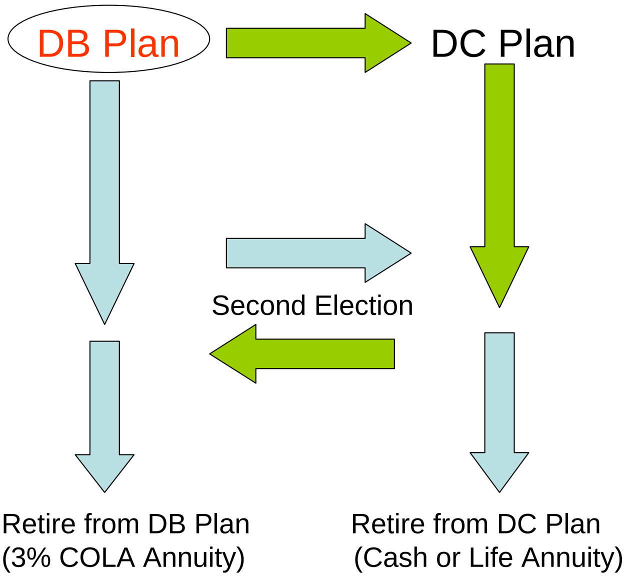
Exhibit #1

The Florida Decision Tree



DB Plan

DC Plan

Second Election

Retire from DB Plan
(3% COLA Annuity)

Retire from DC Plan
(Cash or Life Annuity)

Figure #1

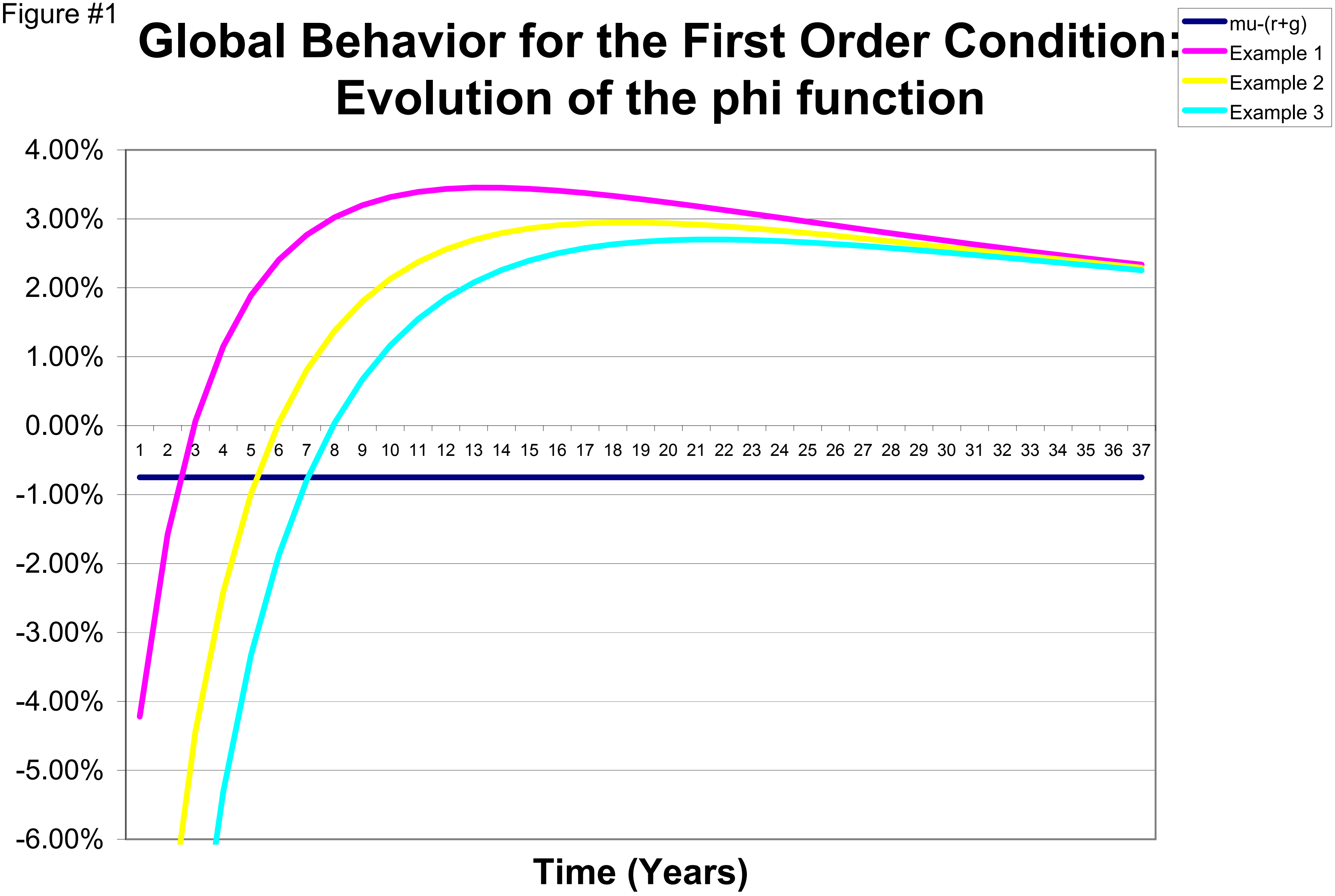

Global Behavior for the First Order Condition:
Evolution of the phi function
mu-(r+g)
Example 1
Example 2
Example 3
4.00%
3.00%
2.00%
1.00%
0.00%
-1.00%
-2.00%
-3.00%
-4.00%
-5.00%
-6.00%
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37
Time (Years)

Figure #2

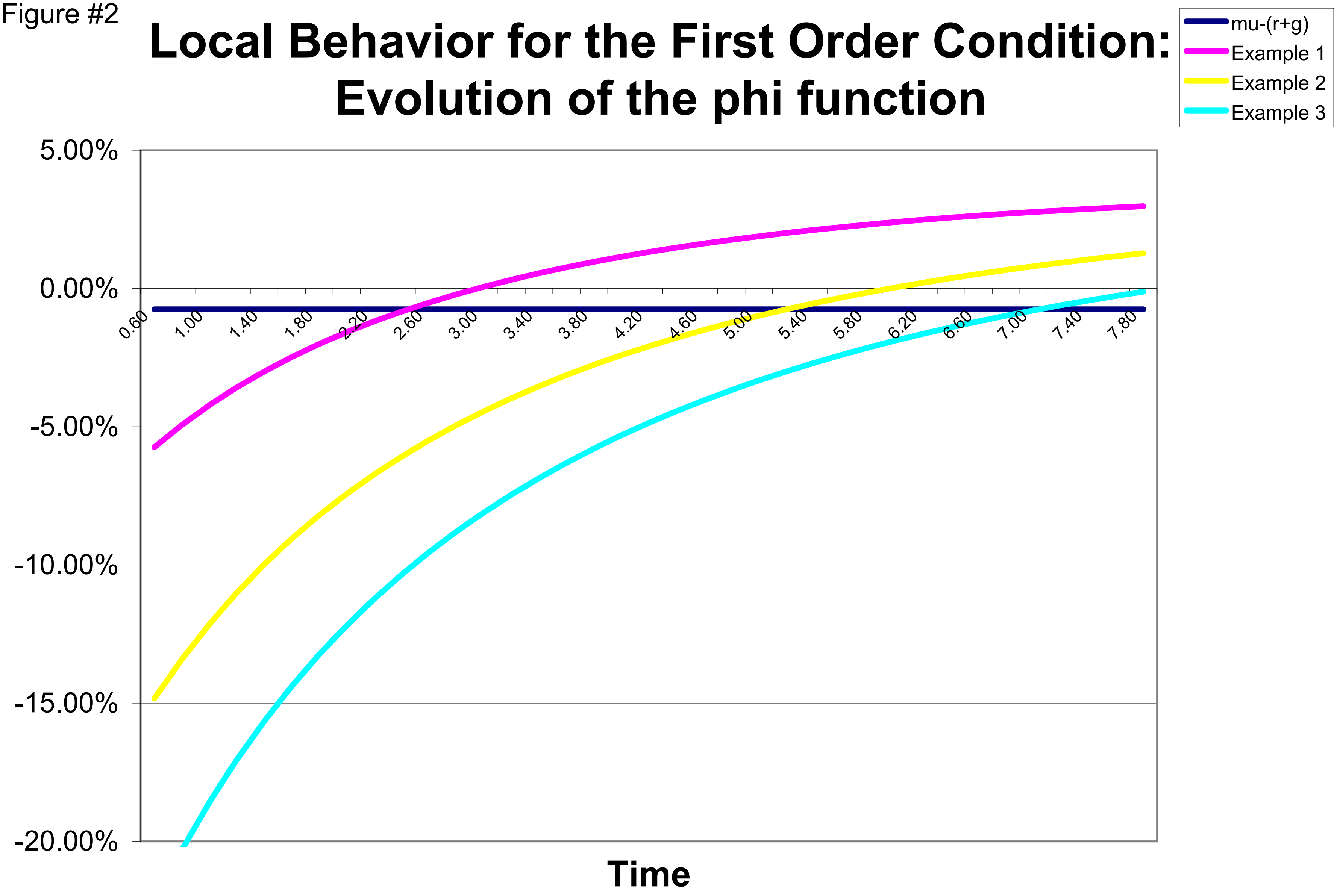

Local Behavior for the First Order Condition: Evolution of the phi function

Table 1

The Threshold Investment Return (mu), Required to Justify Staying in the DC Plan.

Entry Age	*Initial Service*	*Age at which the second election is being contemplated*					
		30	**35**	**40**	**45**	**50**	**55**
50	**5**					15.7%	17.1%
	10					14.3%	15.6%
	15					13.8%	14.9%
	20					13.5%	14.5%
	25					13.4%	14.2%
40	**5**			-5.0%	10.0%	13.8%	14.9%
	10			3.9%	10.1%	13.5%	14.5%
	15			6.8%	11.4%	13.3%	14.2%
30	**5**	-51.3%	-5.4%	6.8%	11.4%	13.3%	14.2%
	10	-19.3%	0.6%	8.3%	11.6%	13.2%	13.9%

Notes: Assuming retirement at age 60, DB rate of 1.6% of salary, and DC rate of 9% of salary

Table #2

Number of Florida State Employees in Various Age Buckets:

Current age….

SVC	62	57	52	47	42	37	32	27	22
42	*127*								
37	*352*	*230*							
32	*955*	*2,458*	*2,602*						
27	*3,611*	*7,431*	*13,852*	*8,407*					
22	*4,056*	*7,664*	*11,444*	*14,581*	*6,860*				
17	*4,640*	*8,235*	*12,320*	*14,099*	*15,040*	*6,897*			
12	*7,743*	*10,613*	*16,505*	*19,395*	*19,425*	*19,796*	*8,321*		
7	*9,645*	*9,473*	*14,429*	*18,076*	*18,751*	*19,139*	*20,685*	*9,457*	
2	*8,216*	*8,737*	*13,450*	*17,304*	*21,548*	*23,278*	*24,921*	*33,312*	*14,654*

Percent of Florida State Employees in Various Age Buckets:

Current age….

SVC	62	57	52	47	42	37	32	27	22
42	*0.02%*								
37	*0.07%*	*0.04%*							
32	*0.18%*	*0.46%*	*0.49%*						
27	*0.68%*	*1.39%*	*2.60%*	*1.58%*					
22	*0.76%*	*1.44%*	*2.15%*	*2.74%*	*1.29%*				
17	*0.87%*	*1.55%*	*2.31%*	*2.65%*	*2.82%*	*1.29%*			
12	*1.45%*	*1.99%*	*3.10%*	*3.64%*	*3.65%*	*3.72%*	*1.56%*		
7	*1.81%*	*1.78%*	*2.71%*	*3.39%*	*3.52%*	*3.59%*	*3.88%*	*1.78%*	
2	*1.54%*	*1.64%*	*2.52%*	*3.25%*	*4.04%*	*4.37%*	*4.68%*	*6.25%*	*2.75%*

Source: Florida State Board of Administration; 2001

Table #3

Average Salary of Florida State Employees in Various Age Buckets:

Current age….

SVC	62	57	52	47	42	37	32	27	22
42	*$ 52,756*								
37	*$ 50,000*	*$ 45,652*							
32	*$ 44,712*	*$ 48,779*	*$ 46,925*						
27	*$ 40,072*	*$ 43,736*	*$ 46,650*	*$ 38,670*					
22	*$ 35,528*	*$ 38,570*	*$ 41,227*	*$ 42,171*	*$ 36,633*				
17	*$ 31,853*	*$ 34,608*	*$ 36,583*	*$ 37,230*	*$ 36,769*	*$ 32,840*			
12	*$ 27,496*	*$ 30,934*	*$ 32,002*	*$ 32,075*	*$ 31,619*	*$ 32,158*	*$ 28,686*		
7	*$ 22,385*	*$ 26,243*	*$ 27,362*	*$ 27,086*	*$ 26,340*	*$ 27,076*	*$ 28,040*	*$ 24,923*	
2	*$ 14,922*	*$ 19,629*	*$ 20,914*	*$ 20,828*	*$ 19,626*	*$ 19,473*	*$ 21,287*	*$ 22,595*	*$ 15,504*

Source: Florida State Board of Administration; 2001

Table #4

Theoretical Option Value for Florida State Employees:

Current age….

SVC	62	57	52	47	42	37	32	27	22
42	*0.0%*								
37	*0.0%*	*0.0%*							
32	*0.0%*	*0.0%*	*0.0%*						
27	*0.0%*	*0.0%*	*0.0%*	*0.0%*					
22	*0.0%*	*0.0%*	*0.0%*	*0.0%*	*0.0%*				
17	*0.0%*	*0.0%*	*0.0%*	*0.0%*	*0.0%*	*0.5%*			
12	*0.0%*	*0.0%*	*0.0%*	*0.0%*	*0.0%*	*1.3%*	*4.6%*		
7	*0.0%*	*0.0%*	*0.0%*	*0.0%*	*0.3%*	*2.8%*	*7.0%*	*12.3%*	
2	*0.0%*	*0.0%*	*0.0%*	*0.0%*	*1.6%*	*5.5%*	*10.6%*	*16.6%*	*23.4%*

Assumptions: 4.75% salary growth rate, and 8% actuarial valuation rate.