



Asset Allocation and Guaranteed Living Benefits in Variable Annuities

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Presentation Outline.

- Most Variable Annuity (VA) policies contain guaranteed living benefit (GLB) riders which protect the account from negative investment returns and/or extreme longevity.
- I will focus in detail on the mechanics of one of these GLB products, namely a **Guaranteed Minimum Income Benefit (GMIB)**.
- Using exclusive and unique data from LIMRA on the behavior of over 500,000 VA policyholders, I will compare their asset allocation choices with a theoretical optimum under a GMIB.

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Why are VAs getting so complicated?

- In the old days Variable Annuities were promoted as tax-sheltered (expensive) mutual funds with (minimal) death benefits.
- Now they are being financially engineered as pension replacements and retirement income supplements.
- At the core of all VA riders is a type of **longevity insurance**...but they are quite heterogeneous in nature and structure.

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Probability of Survival at Age 65

To Age:	Female	Male
70	93.9%	92.2%
75	85.0%	81.3%
80	72.3%	65.9%
85	55.8%	45.5%
90	34.8%	23.7%
95	15.6%	7.7%
100	5.0%	1.4%

Source: Society of Actuaries RP-2000 Table with full projection.

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- *Best-practice female life expectancy...has risen during the last 160 years at a steady pace of almost **3 months per year**...*
- *One reasonable scenario would be that this trend will continue...life expectancy will reach **age 100 in about six decades**.*

“Broken Limits to Life Expectancy”
J. Oeppen and J.W. Vaupel
SCIENCE, Vol. 296, 10 May 2002

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How to explain **Longevity Insurance**

- Five 95 year-old females invest \$100 each in a tontine pool – for one year – and only those living to age 96 will share the money.
- The \$500 is invested at 5% interest for the year.
- Survival rate for a 95 year-old female is 80%
- The \$525 is shared amongst four survivors.
- The \$131.25 is a 31.25% return on investment, which is 26.25% more than the investment rate.
- These **mortality credits** form the basis of all life (payout) annuity contracts.

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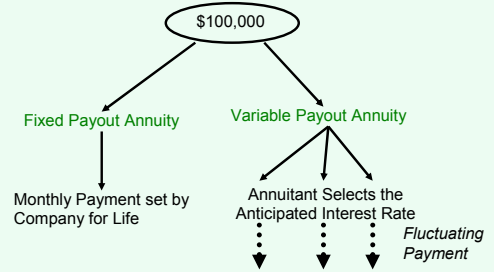
The Older You Are the Greater the Relative Benefit from Annuitization: Unisex Mortality Credits:

Age of Annuitant	Spread Above Pricing Interest Rate (in Basis Points = 1/100 %)	Age of Annuitant	Spread Above Pricing Interest Rate (in Basis Points = 1/100 %)
55	35	80	414
60	52	85	725
65	83	90	1256
70	138	95	2004
75	237	100	2978

Source: The IFID Centre calculations
Assuming 40m/60f (static) Annuity 2000 Table at 6% net interest.



The Payout Annuity Universe



Back to Variable Annuities... ...Major Types of G.L.B.

- Guaranteed Annuity Option (GAO); historically contained in all Variable Annuities.
- Guaranteed Minimum Accumulation Benefit (GMAB); very similar to a long-term equity put.
- Guarantee Minimum Withdrawal Benefit (GMWB) for Life; SWiP with complex put.
- Guarantee Minimum Income Benefit (GMIB); the focus of this presentation...



Option to Annuitize: Type I

$$\frac{W}{a_x} \Rightarrow \text{Lifetime Income}$$

$W := \text{Account Value}$

$a_x := \text{Annuity Factor}$



Option to Annuitize: Type II

$$\frac{W}{\text{Max}[g, a_x]} \Rightarrow \text{Lifetime Income}$$

$W := \text{Account Value}$

$g := \text{Guaranteed Factor}$

$a_x := \text{Annuity Factor}$



Option to Annuitize: Type III

$$\frac{\text{Max}[G, W]}{a_x} \Rightarrow \text{Lifetime Income}$$

$W := \text{Account Value}$

$G := \text{Guaranteed Base}$

$a_x := \text{Annuity Factor}$



Option to Annuitize: Type IV

$$\text{Max} \left[\frac{\text{Max} [G, W]}{f}, \frac{W}{a_x} \right] \Rightarrow \text{L. I.}$$

W := Account Value

G := Guaranteed Base

f := Fixed Factor

a_x := Annuity Factor



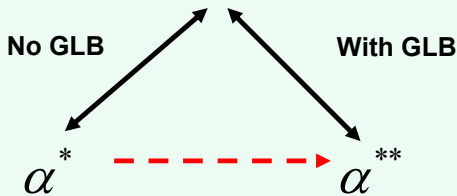
Main Research Question:

- How would you allocate your assets within a variable annuity if you had an option to annuitize in the form of a GMIB?
- Would you take-on greater investment risk because of the downside protection?
- Do VA + GMIB policyholders have greater equity exposure in practice?
- How does asset allocation vary with age?



How to think about the issue...

Age & Risk Aversion



Empirical Evidence (Many Thanks to LIMRA!)

- The next few slides display the actual asset allocations of over 500,000 VA policyholders with and without GMIB riders.
- We have focused on the impact of (i) age and (ii) distribution channel in determining the equity exposure within the VA.
- Notation: High & Medium Risk (HMR) contains equity-based sub accounts, versus Low Risk (LR) which includes bonds, money market & fixed accounts.



Fund / Sub-account Classification: Use Morningstar

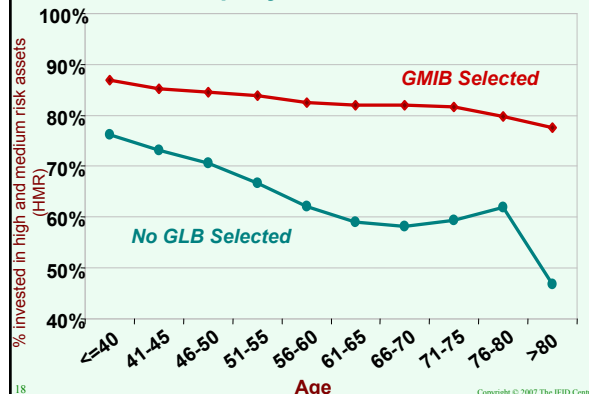
- **HMR:** Small-cap, Mid-cap, Large-cap, International, High Yield and Real Estate:



- **LR:** Bond, Money Market, Fixed Account:

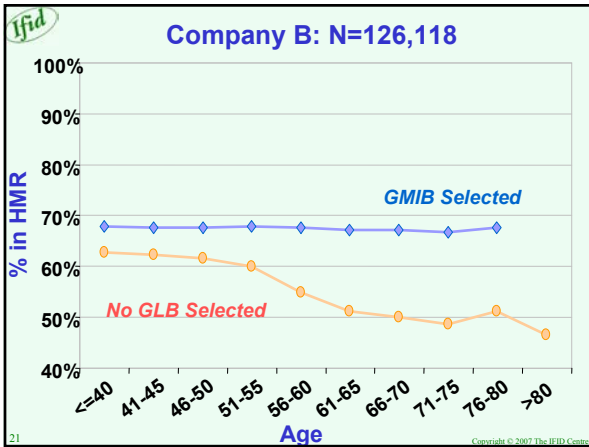
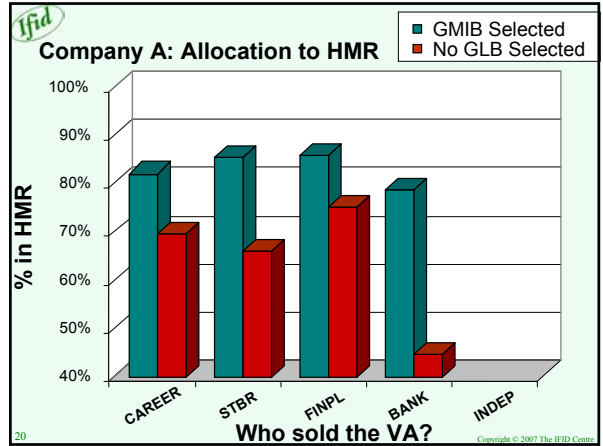


Company A: N=170,462



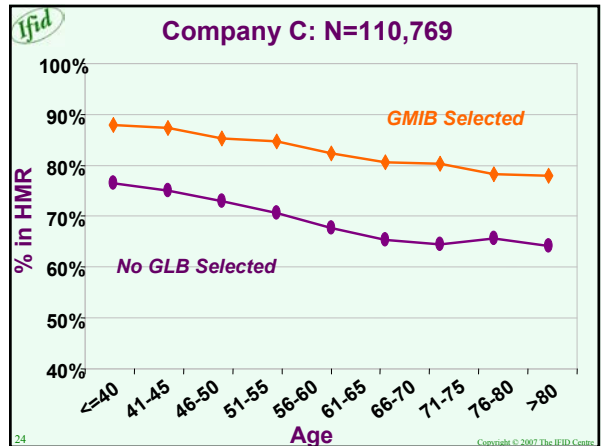
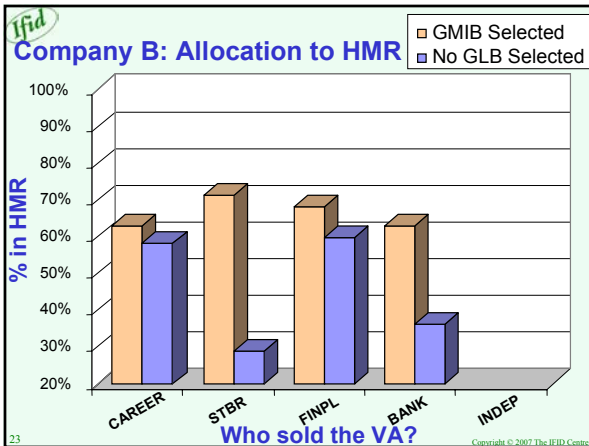
Company A: Allocation to HMR Assets

AGE	% in HMR		Additional HMR Exposure
	GMIB only	no GLB	
<=40	86.9%	76.3%	10.6%
41-45	85.3%	73.1%	12.2%
46-50	84.6%	70.6%	14.0%
51-55	83.8%	66.7%	17.1%
56-60	82.4%	62.2%	20.3%
61-65	82.0%	59.0%	23.0%
66-70	82.1%	58.3%	23.8%
71-75	81.6%	59.3%	22.2%
76-80	79.7%	61.9%	17.8%
>80	77.5%	46.7%	30.8%
Total # of Policies	104,377	66,085	



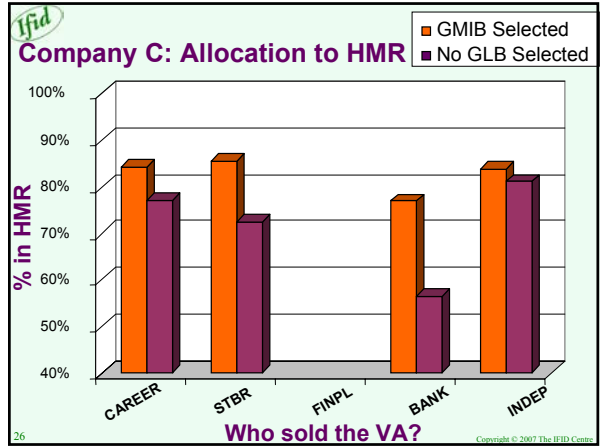
Company B: Allocation to HMR Assets

AGE	% in HMR		Additional HMR Exposure
	GMIB only	no GLB	
<=40	67.8%	62.8%	5.0%
41-45	67.8%	62.2%	5.6%
46-50	67.6%	61.7%	5.9%
51-55	67.9%	59.9%	8.0%
56-60	67.6%	54.9%	12.7%
61-65	67.3%	51.1%	16.1%
66-70	67.1%	50.0%	17.2%
71-75	66.8%	48.7%	18.1%
76-80	67.6%	51.1%	16.4%
>80	N/A	46.4%	N/A
Total # of Policies	89,949	36,169	



Company C: Allocation to HMR Assets

AGE	% in HMR		Additional HMR Exposure
	GMIB only	no GLB	
<=40	88.1%	76.7%	11.4%
41-45	87.4%	75.1%	12.3%
46-50	85.2%	73.1%	12.2%
51-55	84.7%	70.9%	13.9%
56-60	82.4%	67.8%	14.6%
61-65	80.7%	65.4%	15.3%
66-70	80.3%	64.5%	15.8%
71-75	78.3%	65.7%	12.6%
76-80	78.1%	64.2%	13.8%
>80	N/A	N/A	N/A
Total # of Policies	38,674	72,122	



Pooled Regression Results:
Dependent Variable = HMR% in VA

Variable	With GMIB		Without GLB	
	Estimate	t-value	Estimate	t-value
Male	+0.0123	16.4	+0.0256	18.24
Age	-0.0015	-43.09	-0.0041	-76.1
IRA	-0.0044	-5.59	-0.0098	-6.37
STBR	+0.0112	2.6	+0.0296	7.27
BANK	-0.0576	-13.02	-0.1433	-34.42

N = 368K (with GMIB), 272K (w/o GMIB), Adj. R-sq = 0.15

Takeaway...

- We find that policyholders increase their risk exposure by up to 20 to 25 percentage points. This is not limited only to younger investors.
- There is a strong distribution channel effect, which might relate to clientele as opposed to the sales process.
- Remember that in many cases there are asset allocation restrictions (i.e. model portfolios) when the GMIB rider is selected. In other words, some of this effect might not be completely voluntary.

Theoretical Model...


- In the fine tradition of dynamic portfolio choice and modern portfolio theory I will now derive an optimal asset allocation in the presence of a GMIB.
- Think of this as a normative theory of how you should change your asset allocation if you are suddenly granted a put option.
- This is more complicated than you think!

Assumptions & Disclaimers

- You face a classic choice between risky and risk-free investment assets.
- You reveal your "natural" risk preference with your initial asset allocation.
- You are then offered a GMIB put-option that is fairly (No Arbitrage) priced.
- The embedded guaranteed annuity factor is competitive with market rates.
- You understand annuitization, but are subject to liquidity shocks...
- How do you change your mix?

The Model

$$\gamma \Leftrightarrow \max_{\alpha} E[u(W)] \Leftrightarrow \alpha^*$$

$$\gamma \Leftrightarrow \max_{\alpha} E[u(\max[W, K])] \Leftrightarrow \alpha^{**}$$


**Option Strike Price =
Annuitized Value of Guaranteed Income**

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Optimal Allocations & GMIB: Age 55

Natural HMR	Low (10%) Lapse Rate	(50%) Medium Lapse Rate	(90%) High Lapse Rate
20%	47%	26%	20%
40%	52%	48%	40%
60%	69%	65%	60%
80%	87%	83%	80%
90%	94%	92%	90%

32 Assumptions: 15 year horizon, 5% guarantee, No age set-back. Copyright © 2007 The IFID Centre

Ok, so are the actual asset allocations we observe, justified?

- In theory yes, but...
- Remember, we did NOT model any particular company's GMIB. Our analysis is generic.
- That said, at higher ages it is hard to justify an extra 15 percentage points of equity risk.
- The (i) assumed lapse rates, (ii) mortality table set-backs, and (iii) pricing, play an enormous role.
- In practice, if the strike price of the embedded option is (very) out-of-the money, the optimal allocation to HMR (equity) might not change.

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Final Takeaway Points:

- The VA+GLB industry is growing in importance. It can be measured in hundreds of billions of dollars and should not be ignored by traditional investment-focused analysts.
- The embedded put options are extremely complicated. Generalizations regarding value and optimal behavior are quite difficult.
- One thing is for certain, VA policyholders are taking on more equity-market risk, which I believe is partially – although perhaps not fully - justified by rational asset allocation models.

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