Personal Portfolio Allocation Using Actuarial Concepts

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Agenda for Tonight

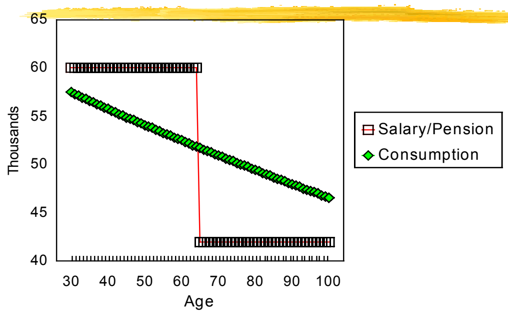
- Economic Lifecycle Theory
- A Model of Risk and Return
- Space & Time Diversification
- Using Probabilities to Analyze:
  - Dollar-Cost Averaging
  - Optimal Asset Allocation
- Conclusions and Final Thoughts

A Thought Experiment: Assume The Future is Known

- Work 35 years & Retire 35 years.
- Live in good health to age 100.
- Salary & pension never change.
- All money earns 1% per year.
- No inflation.
- Question: What would personal financial planning look like?

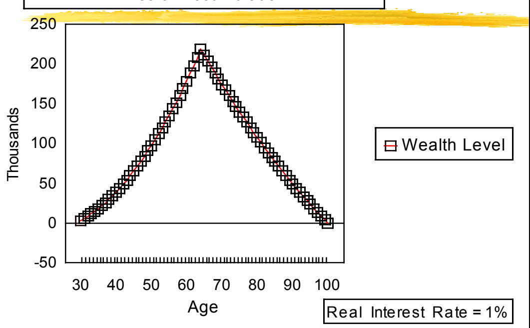
Simple Life Cycle Model

35 Working Years & 35 Retirement Years



Simple Life Cycle Model

Wealth Accumulation



Pre-Retirement: Equation of Wealth Management

$$W_n = W_0(1+k)^n + \sum_{i=1}^{n} (E_i - C_i)(1+k)^{n-i}$$

Post-Retirement: Equation of Wealth Management

$$W_n = \sum_{i=n+1}^{d} \frac{C_i}{(1+k)^{i-n}}$$

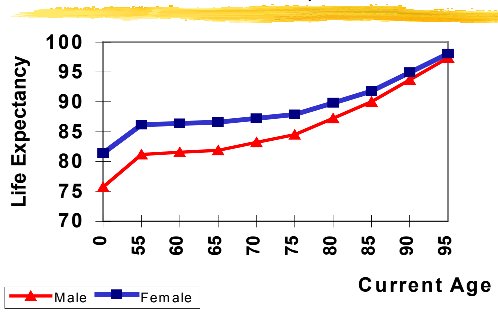
Two Greatest Sources of Uncertainty during the Life-Cycle

- Mortality: Health plus Life Span

and

- Investment: Rates of Return

How Long Do People Live? At birth: M = 75.7, F = 81.4



Risk and Return:

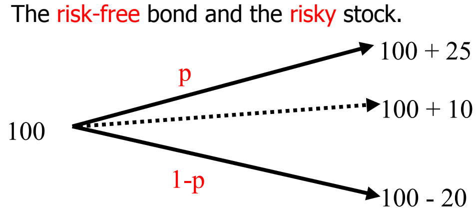
Higher Return $\xrightarrow{\text{Yes}}$ **Higher Risk**

Higher Risk $\xrightarrow{\text{Not Necessarily}}$ **Higher Return**

No compensation for "silly" risk taking.

Why do people buy risky stocks?

A choice of two assets:
The risk-free bond and the risky stock.



A Utility Analysis:

- CLAIM: If you invest in the risky asset, it is because you get more utility compared to investing at the risk free rate.
- We can compute the threshold probability for investing in the risky asset.

Take the Risk?

$$U(WR) < E[U(W\tilde{X})]$$

In our case:

$$\ln[110] <$$
$$p\ln[125] + (1-p)\ln[80]$$

Condition for Investing:

$$p > \frac{\ln[110] - \ln[80]}{\ln[125] - \ln[80]}$$
$$= 0.7135$$

Risk Averse People Choose to Invest..

- Only if they believe the probability of winning is high enough.
- Only if they expect the risky return to beat the risk-free rate.
- Only if they are compensated for the downside risk.
- Only if they expect the risk premium to be positive.

More General Utility

$$U(w) = \frac{1}{1-\gamma} w^{1-\gamma}$$

Condition for Investing

$$p > \frac{(115/80)^{(1-\gamma)} - 1}{(125/80)^{1-\gamma)} - 1}$$

gamma	Pr(Gain)	Ex[Gain]
8	0.933	$22.01
4	0.833	$17.52
3	0.797	$15.90
2	0.757	$14.09
1	0.713	$12.11
0	0.666	$10.00
-2	0.568	$5.57

Equity Valuation

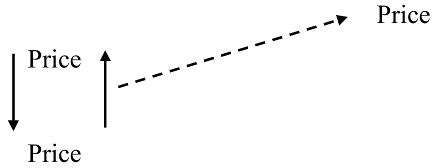
- The value of a stock should be equal to the future stream of dividends discounted at a risk-adjusted rate:

$$V = \sum_{i=1}^{\infty} \frac{D_i}{(1 + R + \psi)^i}$$

- If the expected risk-premium declines, the present value increases, which increases the stock price.

Appetite for Risk and Stock Prices

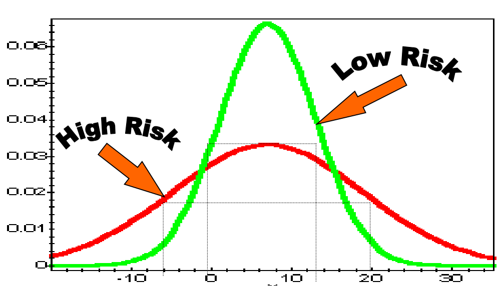
Now..............................Then



Risk and the Stock Market

- My hypothesis: Investors are becoming more accustomed to stock market risk.
- As the risk aversion of the population declines, the value of stock prices increases because they are willing to accept a lower risk premium in compensation for the uncertainty.
- In the future, prices will not rise as fast!

The Statistics of Investing



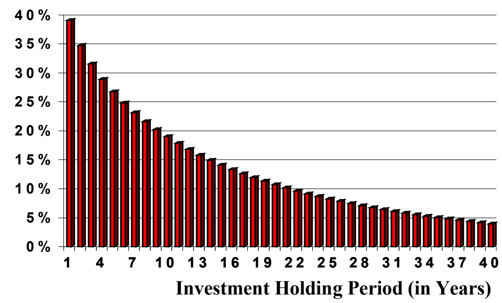
What is Financial Risk?

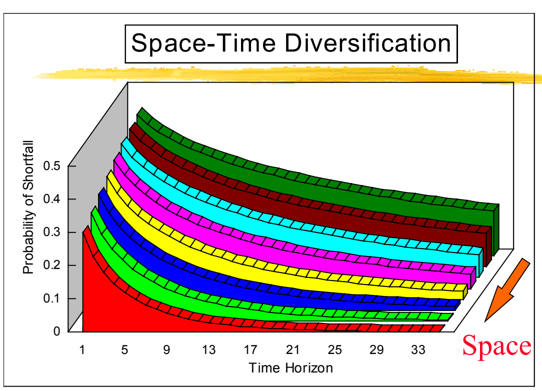
- RISK =
  Variability and Fluctuations?

OR

- RISK =
  Probability of Regret?

The Probability of Regret: Equities do worse than T.Bills





Diversification over Time & Space:

- Risk has a Dimension of Time
- Shortfall Decreases Exponentially
- Global Investing is Crucial
- Allocation Should not be Static
- Day-to-Day Volatility is Meaningless
- More Time = More Aggressive

10 Minute Break

- …

Probability Example: Saving for Retirement

- You want to retire in 25 years with a 'nest egg' of $250,000 - in real terms - i.e. in today's $.
- How much money do you have to save every month - in real terms - in order to assure a 90% chance that you will reach your retirement financial goal?
- **ANSWER**: It depends on investment mix.

Each month you must save...

100% T.Bills	50% Equity + 50% T.Bills	100% Equity
$685	$530	$405

to assure a 90% chance of having $250,000 in 25 years

Source: HMR/97

Asset Allocation Example Retirement Question:

- You have just retired at age 60.
- You have a $100,000 nest egg.
- You want to consume $7,500 per year, in real terms, for the rest of your life.
- Is your standard of living sustainable, with a wealth-to-consumption ratio of 13.3/1?
- ANSWER: It depends on investment mix!

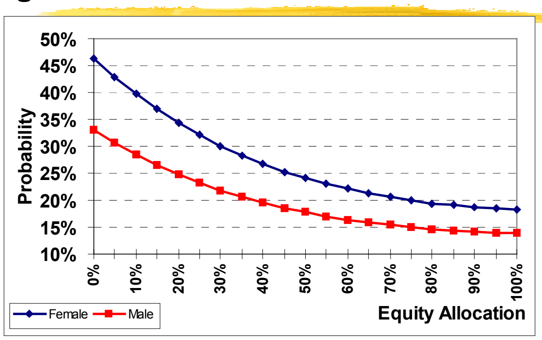
Probability of Lifetime Ruin Case #1: Age=60, W/C=13.3

Stock-Equity	Female	Male
0%	46%	33%
25%	32%	23%
50%	24%	18%
75%	20%	15%
100%	18%	14%

The Remainder in T.Bills.

Source: HMR/97

What is the probability of starvation?
Age =60: W/C=13.3



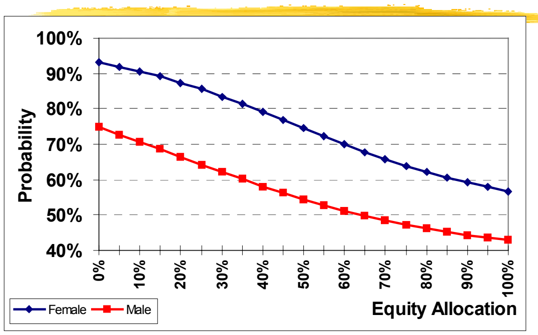
Probability of Lifetime Ruin
Case #2: Age=65, W/C=10

Stock-Equity	Female	Male
0%	93%	74%
25%	85%	64%
50%	74%	54%
75%	63%	47%
100%	56%	42%

The Remainder in T.Bills.

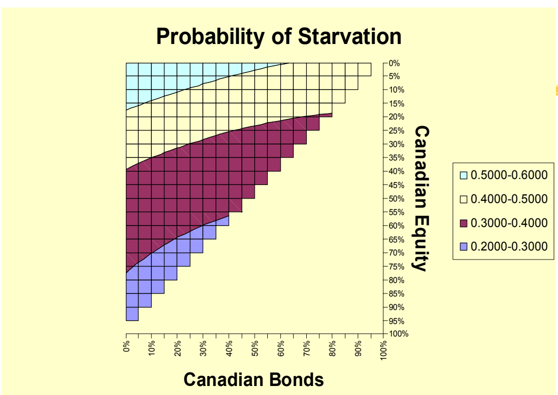
Source: HMR/97

What is the Probability of Starvation?
Age =65: W/C=10



Equity vs. Bonds What are the Odds?

- Male Age 65.
- $100,000 to invest.
- $7,500 to consume.
- What is best mix between Equity, Bonds and T.Bills, to minimize the probability of Shortfall?
- This is a 2+1 Dimensional Problem



Example: DCA vs. LSI

- Is it better to invest in one lump-sum or should you dollar-cost average, if you have the money now?
- What is the probability of regret from following each of the two strategies?
- CLAIM: It is better to split the money in two, and buy-and-hold, compared to dollar-cost averaging.

How does DCA Work?

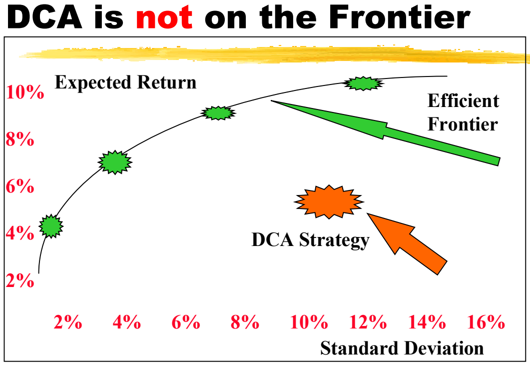
Invest $100 every quarter

Date	Price	# of Units
Jan. 1.	$100	1
Apr. 1.	$50	2
Jul. 1.	$200	0.5
Oct. 1.	$25	4
Average:	$93.75	$400/7.5 = $53.3

Dollar-Cost Averaging vs. Lump-Sum Investing

Equity Fund	Expected Wealth	Variability
$10,000	$11,000	$1,500
$5,660	$10,783	**$850**
$4,960	**$10,748**	$744
$0	$10,500	$0
DCA	**$10,748**	**$850**

DCA is not on the Frontier



Conclusion on DCA.

- Does not reduce volatility effectively.
- Replacing a large risk, with many smaller sequential risks, does not reduce risk.
- Better to pick an asset allocation you are comfortable with and stick to it.
- Contrast the Logic with Emotion.

Final Words

- The common theme of the last few weeks is that when discussing RISK we must employ concepts and ideas from both financial economics and actuarial science.
- In the future, both these fields will and must come together to better analyze and manage the risks we face in our life.
- Thank you for inviting me!

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