

**Optimal Annuitization Policies:  
Analysis of the Options**

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**Fixed Immediate Life Annuities**

**Agenda:**

- Briefly describe the Canadian environment.
- Problem: To annuitize or not to annuitize?
- Discuss historical returns from annuities.
- Present a 'model' for optimal annuitization.
- Give practical advice and conclusions.

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**Canadian Saving for Retirement**

- The savings vehicle of choice is the RRSP.
- Contributions are tax-deductible and gains are tax-deferred until retirement.
- At age 69 Canadians must either:
  - collapse the RRSP (taxable event)
  - buy a life annuity
  - convert to a withdrawal plan (RRIF)
- This impacts 200,000 Canadians annually.

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## To Annuitize or Not To Annuitize?

- The annuity provides no bequest or estate.
- Guarantees and refunds reduce benefits.
- The annuity is a non-reversible investment.
- A life annuity is a 'fixed income' product.
- It provides longevity insurance.

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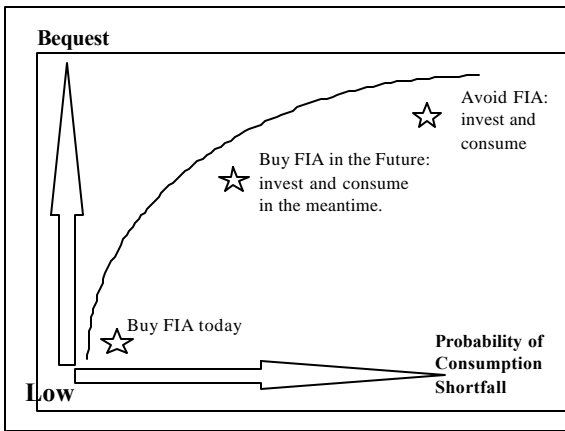
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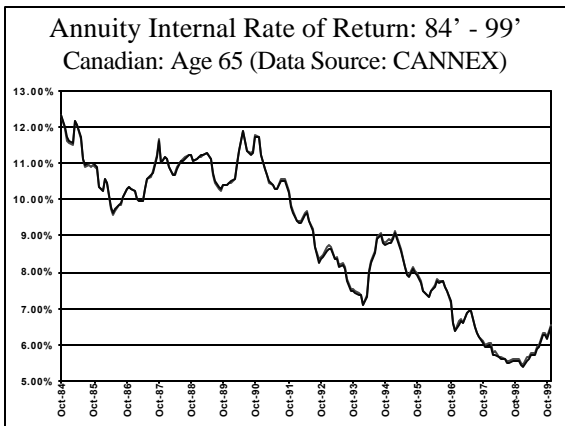
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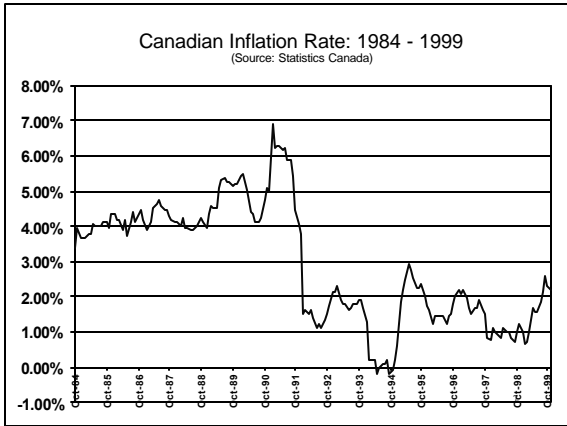
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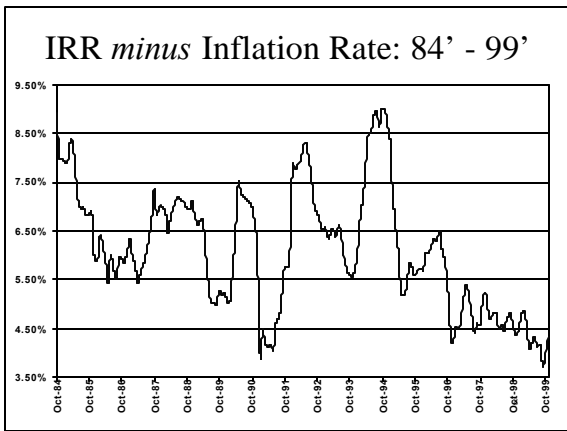
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Basic Life Annuity Pricing

$$a_x = (1+l) \sum_{i=1}^{\infty} \frac{i P_x}{(1+R)^i}$$

“..Price is equal to actuarial present value multiplied by one-plus a load factor...”

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**What is the value of a life annuity?**

- We can compare the expected P.V. of payments to the actual price of the annuity.
- This should give us a measure of the return as well as fair pricing.
- Comparing population vs. annuitant mortality tables will provide an estimate of the adverse selection costs.
- See: "New Evidence on the Money's Worth of Individual Annuities", *American Economic Review*, 2000

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**Value per Dollar of Annuity  
Canadian Market: 1985-1998**

Mortality		Gov. Rate	Corp. Rate
Annuity:	Male	0.958	0.903
	Female	0.953	0.894
Population:	Male	0.912	0.866
	Female	0.924	0.872

Assumptions: Full Term Structure  
Mortality: IAM 1983 with Scale G adjustment  
*Journal of Risk and Insurance*, Sep. 1998

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**Can you 'beat' the rate of return  
from a life annuity?**

**Fact:**

$$a_x(1 + K) - 1 > a_{x+1}$$

**If:**

$$K \geq K^* = \frac{\overline{a_{x+1}}}{a_x} + \frac{T}{a_x} - 1$$

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You can beat the annuity  
with a rate-of-return:

$$K^* = \frac{1}{{}_1p_x} (1 + R) - \frac{l}{a_x} - 1$$

13

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**Numerical Example:  
How To Beat a Life Annuity**

Age	Death Probability	Required Return
55	2.26/1000	6.2%
65	5.76/1000	6.6%
75	16.34/1000	7.8%
85	54.05/1000	12.1%
90	95.84/1000	17.2%

Assumptions: R=6%, load=0%, IAM1996 Table

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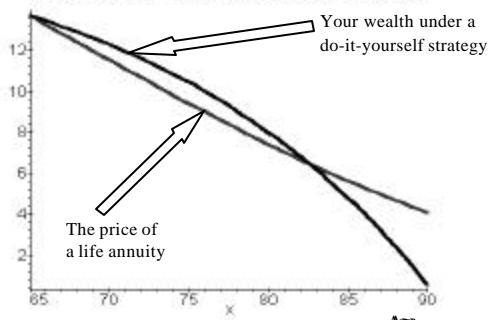
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Evolution of Wealth v.s. Cost of Annuity



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## What if Returns are Random? Monte Carlo Simulation

- Simulate interest rates and equity market returns, using historical frequencies.
- Compute the probability of 'beating the return' from the life annuity.
- Locate the 'optimal' age at which to annuitize, by fixing a tolerance level.

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## Probability of Beating the Annuity Age 65: Simulation Results

Wait...	Male	Female
5 Years	77.1%	78.3%
10 Years	79.5%	81.6%
15 Years	81.3%	84.1%
20 Years	83.9%	85.3%

Assumptions: Interest Rate = 7%, Load = 10%  
Assets grow at 13% with Volatility of 17%  
Mortality: IAM 1996 with Scale G adjustment

17

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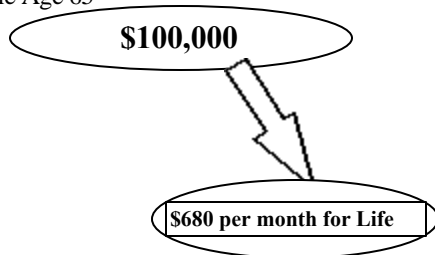
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## Fixed Immediate Annuity

Female Age 65



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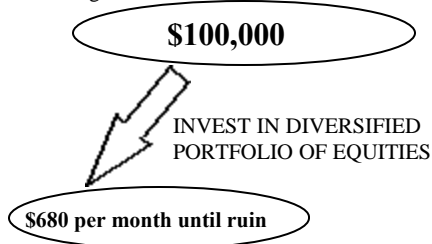
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## Do-It-Yourself Annuity

Female Age 65



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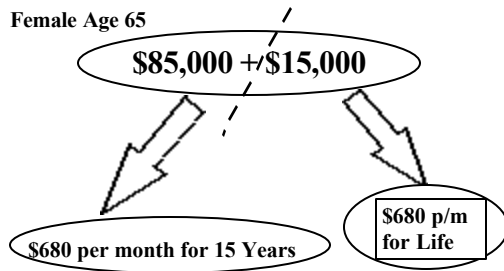
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## Do-it-yourself and then switch

Female Age 65



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## Main Conclusions:

- Annuities provide valuable longevity insurance and should be marketed as such. They are not just fixed income products.
- One can easily beat the return from an FIA prior to age 75. It is best to wait!
- The ideal product is a real deferred annuity that starts at age 80-85.
- Variable Immediate Annuities -- which are currently not available in Canada -- should grow in popularity.

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### **How to Reach Me:**

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22

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