Betting on Death and Capital Markets in Retirement:

A Shortfall Risk Analysis of Life Annuities
versus Phased Withdrawal Plans

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Toronto
Three Uncertainties in Retirement:
A Financial Perspective

Investment Returns?

Investment horizon?

Bequest?

Source: Die Zeit
„Rente“=Retirement / „Ziel“=Goal
Motivation

• Compared to accumulation phase:
  ➢ Uncertainty about capital markets
  ➢ Uncertainty about investment horizon

• Interest in alternative payout designs:
  ➢ Risk-return tradeoffs: Benefits, shortfalls, and bequests
  ➢ Incorporate asset allocation and withdrawal rules

• Importance:
  ➢ 1st pillar state pensions in decline, more DC plans
  ➢ Retirees responsible for decumulation phase
  ➢ UK, Germany require mandatory annuitization (75/85)
Phased Withdrawal Plans

- Retirement assets invested in Individual Pension Account
  - Asset Allocation

- Retiree consumes from the IPA periodically
  - Withdrawal Rule

- Advantages compared to Life Annuity
  - High flexibility, liquidity
  - Bequest potential
  - Higher benefits

- Risks of Phased Withdrawal Plans
  - Lower benefits than Life Annuity → Consumption Shortfall
  - Longevity risk (No risk pooling) → “Betting on Death”
  - Capital market risk → “Betting on Capital Markets”
Phased Withdrawal Plans

Types of Withdrawal Plans

Fixed Withdrawals
- constant
- increasing
- decreasing
Amount in EURO

Asset Allocation
- Stocks
- Bonds
- Cash
- Mixed

Variable Withdrawals
- constant
- increasing
- decreasing
Benefit-to-wealth ratio
Fixed Withdrawal Plan

Retiree has sum of money $V_0$

- invested in financial assets earning returns $R_t$.
- Each period, he consumes $B$ equal to the life annuity as long as possible:

$$B_t = \min(B, V_t).$$

- Non-linear Intertemporal budget constraint:

$$V_{t+1} = (V_t - B_t)(1 + R_t) = \begin{cases} (V_t - B)(1 + R_t) & V_t > B \\ 0 & V_t \leq B. \end{cases}$$

$\Rightarrow$ Consumption risk = fund exhaustion while still alive
Variable Withdrawal Plans

• Plan pays an *ex ante* specified fraction $\omega_t$ of remaining retirement funds [e.g. 5%].

$$B_t = \omega_t \cdot V_t$$

• Linear Intertemporal budget constraint:

$$V_{t+1} = (V_t - B_t) \cdot (1 + R_t) = (1 - \omega_t) \cdot V_t \cdot (1 + R_t)$$

➡️ Consumption risk = lower benefits than benchmark while still alive
Specific Variable Withdrawal Rules

“Fixed Percentage” withdrawal rule:
- Constant and fixed fraction \( \omega = \omega_0 = \omega_1 = \ldots = \omega_t \).

"1/T Rule" withdrawal rule:
- Withdrawal fraction set to maximum possible plan duration \( T \)

\[ \omega_t = \frac{1}{T - t}. \]

“1/E[T(x)]" withdrawal rule:
- Withdrawal fraction determined by retiree’s remaining life expectancy

\[ \omega_t = \frac{1}{E[T(x + t)]}. \]
The Benchmark Life Annuity

• Characteristics
  ➢ Constant (real) annuity payments until death
  ➢ Offered by commercial insurance companies
  ➢ **Pro:** Pooling of longevity risk / mortality “spread”
  ➢ **Con:** No bequest potential, low flexibility

• Present Relevance
  ➢ Thin private annuity markets around the world
  ➢ Also countries with substantial DC-pension plans
Life Annuity Benefits: Using German / US data

<table>
<thead>
<tr>
<th>Mortality Table</th>
<th>Male</th>
<th>Female</th>
</tr>
</thead>
<tbody>
<tr>
<td>Retirement Age</td>
<td>Life Annuity $ (€) p.a.</td>
<td></td>
</tr>
<tr>
<td>65</td>
<td>5.83 (5.82)</td>
<td>5.22 (5.02)</td>
</tr>
<tr>
<td>70</td>
<td>7.00 (7.03)</td>
<td>6.22 (5.99)</td>
</tr>
</tbody>
</table>

Parenthesis: Results for German Annuity

Immediate Annual Life-long Real Annuity Benefits per EUR 100 Single Premium: Total Expense Loadings 2.785% for Germany; 1% for US; (Real) Discount Factor 1.5%; German DAV R 94 annuitant mortality table (max. age 110); US 2000 basic annuitant mortality table (max age 115)

➢ Mortality “drag” at the cost of no bequest potential
Historical Analysis: Retire in 1957 (German-Case)

Historical Benefits of Withdrawal Plans Conditional on Survival
(60% Equities / 40% Bonds): Life Annuity Benchmark

- Fixed Benefits (=Annuity)
- $1/T$
- $1/E[T]$
- Fixed Fraction Rule (= 5.82%)

Life Annuity Benefits
Historical Analysis: Retire in 1957 (US-Case)

Historical Benefits of Withdrawal Plans Conditional on Survival
(60% Equities / 40% Bonds): Life Annuity Benchmark

Life Annuity Benefits

- Fixed Benefits (=Annuity)
- 1/T
- 1/E[T]
- Fixed Fraction Rule (= 5.83%)
Research Approach

• Evaluate these different strategies against life annuity benchmark
• Stochastic Model (mortality / investments)
• Possible objective functions
  
    – Only look at shortfall probability
    – Only examine withdrawal plans with fixed benefits
  
  ➢ Specific utility functions (Blake, Campbell/Viciera)
    – Must assume exact risk preferences, but…
Our Contributions

➡️ Using risk value models:

- Our risk measure incorporates both probability and size of loss
- Compare fixed with different variable withdrawal rules
- Optimize asset allocation
- Optimize design parameters of variable payment schedule
- Study portfolios of withdrawal plans and annuities
Shortfall Risk and “Return” Measures:

**Risk**

- Shortfall Probability
  \[ SP = P(B_t < z) \]

- Mean Excess Loss
  \[ MEL = E(z - B_t \mid B_t < z) \]

- Expected Shortfall
  \[ SE = E[\max(z - B_t, 0)] = SP \times MEL \]

where

- \( B_t \) = benefit of the withdrawal plan
- \( z \) = benefit of the benchmark life annuity

**Return**

- Expected Benefit
  \[ E[B_t] \]

- Expected Bequest
  \[ E[V_t] \]
Withdrawal plans: Risk-Minimizing Investment Allocation

• Objective function:

\[ EPV_{\text{Shortfall}} = \sum_{t=1}^{T} t \cdot p_x \cdot E[\max(z - B_t, 0)] \frac{1}{(1 + r)^t} \]

• This risk measure accounts for:
  ✓ Mortality risk
  ✓ Time preferences
  ✓ Risk preferences for investment uncertainty

• Vary investment mix and withdrawal fraction to minimize Expected PV of Shortfall
Optimized Withdrawal Rules in Risk-Return Context

- EPV_Benefits reflects expected present value of benefit payments conditional on survival:

\[
EPV_{Benefits} = \sum_{t=1}^{T} \frac{p_x E(B_t)}{(1+r)^t}
\]

- EPV_Bequest measures expected present value of inheritance the retiree passes to heirs in the event of death:

\[
EPV_{Bequest} = \sum_{t=1}^{T} \frac{p_x q_{x+t} E(V_t)}{(1+r)^t}
\]
Methodology

- We model withdrawal plans: age 65 to 110 (115)

- Benchmark Annuity
  - US / German Mortality Tables
  - Assumptions about loadings

- Stochastic Model
  - Price dynamics: GBM
  - 1967-2002 yearly real returns
    - German Data
    - US-Data from Ibbotson
  - 100,000 alternative paths for fixed withdrawal plans
    - (Alternative: IG-Approximation accord. Milevski et al.)
  - Analytical closed form solution for variable withdrawal plans
## Optimization Results:
### “Stand Alone Withdrawal Rules” (German case)

### Benefits from Withdrawal Plan

#### Results for Male (Retirement Age 65):

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<tr>
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<th>EPV Shortfall</th>
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<th>EPV Bequest</th>
<th>Investment Weights (in %)</th>
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<tr>
<td></td>
<td></td>
<td>Equity</td>
<td>Bonds</td>
<td>Cash</td>
</tr>
<tr>
<td>Real Annuity €5.82</td>
<td>0</td>
<td>97.29</td>
<td>0</td>
<td>20 80 0</td>
</tr>
<tr>
<td>Fixed Benefit = €5.82</td>
<td>3.58</td>
<td>93.41</td>
<td>53.19</td>
<td>30 70 0</td>
</tr>
<tr>
<td>Fixed Pct. = 5.82%</td>
<td>12.58</td>
<td>92.53</td>
<td>66.06</td>
<td>50 50 0</td>
</tr>
<tr>
<td>1/T Rule Age 110</td>
<td>34.95</td>
<td>82.68</td>
<td>134.41</td>
<td>50 50 0</td>
</tr>
<tr>
<td>1/E(T) Rule</td>
<td>8.27</td>
<td>103.08</td>
<td>39.80</td>
<td>20 80 0</td>
</tr>
</tbody>
</table>

Benchmark Real Life Annuity €5.82 p.a./ €100

*Note: The table entries are EPV values in €.*
### Impact of Mandatory Switching into a Life Annuity at Age 85 (German Case)

**Results for Male (Retirement Age 65 Switching Age 75):**

**Benchmark Real Life Annuity €5.82 p.a./ €100**

<table>
<thead>
<tr>
<th>Strategy</th>
<th>EPV Shortfall</th>
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<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Equity</td>
</tr>
<tr>
<td>Real Annuity €5.82</td>
<td>0</td>
<td>97.3</td>
<td>0</td>
<td>15</td>
</tr>
<tr>
<td>Fixed Benefit until 85</td>
<td>2.8</td>
<td>103.4</td>
<td>33.5</td>
<td>15</td>
</tr>
<tr>
<td>Fixed Pct. Opt $\omega=7.4%$</td>
<td>7.4</td>
<td>108.8</td>
<td>32.3</td>
<td>25</td>
</tr>
<tr>
<td>1/T Rule Opt Age 88</td>
<td>9.5</td>
<td>108.3</td>
<td>35.1</td>
<td>20</td>
</tr>
<tr>
<td>1/E(T) Rule</td>
<td>5.4</td>
<td>104.1</td>
<td>31.2</td>
<td>15</td>
</tr>
</tbody>
</table>
## Portfolio of Phased Withdrawal Plan and Deferred Life Annuity starting at Age 85

### Results for Male (Retirement Age 65 Switching Age 75):
Benchmark Real Life Annuity £5.82 p.a./£100

<table>
<thead>
<tr>
<th>Strategy</th>
<th>EPV Shortfall</th>
<th>EPV Benefits</th>
<th>EPV Bequest</th>
<th>Investment Weights (in %)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Real Annuity 5.828</td>
<td>0</td>
<td>99.0</td>
<td>0</td>
<td>Equity</td>
</tr>
<tr>
<td>Fixed Payment until 85</td>
<td>5.3</td>
<td>100.0</td>
<td>34.4</td>
<td>50</td>
</tr>
<tr>
<td>Fixed Perc. opt. 9.1%</td>
<td>13.4</td>
<td>110.1</td>
<td>33.7</td>
<td>79</td>
</tr>
<tr>
<td>1/T-Rule (T=84)</td>
<td>10.0</td>
<td>110.2</td>
<td>21.2</td>
<td>50</td>
</tr>
<tr>
<td>1/E(T)-Rule</td>
<td>14.6</td>
<td>111.9</td>
<td>37.7</td>
<td>68</td>
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</tbody>
</table>
## Comparison US vs. German Data

<table>
<thead>
<tr>
<th>Rule</th>
<th>Risk</th>
<th>Benefits</th>
<th>Bequest</th>
<th>Equity Exposure</th>
<th>Withdrawal Fraction</th>
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</thead>
<tbody>
<tr>
<td><strong>Fixed Benefits</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>- Stand Alone</td>
<td>++</td>
<td>-</td>
<td>++</td>
<td>++</td>
<td>++</td>
</tr>
<tr>
<td>- Switching (85)</td>
<td>++</td>
<td>++</td>
<td>+-</td>
<td>++</td>
<td>++</td>
</tr>
<tr>
<td>- Deferring (85)</td>
<td>++</td>
<td>-</td>
<td>++</td>
<td>++</td>
<td>++</td>
</tr>
<tr>
<td><strong>Fixed Fraction</strong></td>
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</tr>
<tr>
<td>- Stand Alone</td>
<td>+</td>
<td>++</td>
<td>-</td>
<td>++</td>
<td>+-</td>
</tr>
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<td>- Switching (85)</td>
<td>+</td>
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<td>0</td>
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<td><strong>1/T-Rule</strong></td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>- Stand Alone</td>
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<td>0</td>
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<td>- Switching (85)</td>
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<td>+</td>
<td>0</td>
<td>++</td>
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</tr>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
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<td>+</td>
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<td>++</td>
<td></td>
</tr>
</tbody>
</table>

++ (--): Substantial Higher (Lower) compared with German Data  
+ (-): Higher (Lower) compared with German Data  
++ (+-): slightly higher (Lower) compared with German Data  
0: no change compared with German Data
Conclusions

• Phased withdrawal plans offer many advantages: flexibility, bequests, and possibly higher consumption than life annuities.

• Yet a phased withdrawal plan also requires that attention be devoted to asset allocation and withdrawal rules.

• To minimize the shortfall-risk of consuming less than a real annuity benchmark, retirees should invest their assets more in fixed income than in equities.

• For a fixed withdrawal rule compared to no annuity:
  – Mandatory deferred annuitization and/or a switching rule can enhance expected payouts & cut expected shortfall risk
  – But at cost of reduced bequests.
BACKUP
Optimization Results: “Stand Alone Withdrawal Rules” (US case)

Benefits from Withdrawal Plan

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<td></td>
<td></td>
<td></td>
<td></td>
<td>Equity</td>
</tr>
<tr>
<td>Real Annuity €5.83</td>
<td>0</td>
<td>99.0</td>
<td>0</td>
<td>60</td>
</tr>
<tr>
<td>Fixed Benefit = €5.83</td>
<td>7.0</td>
<td>91.6</td>
<td>72.4</td>
<td>75</td>
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<tr>
<td>Fixed Pct. = 5.83%</td>
<td>14.7</td>
<td>106.9</td>
<td>46.1</td>
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<td>1/T Rule Age 87</td>
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<td>105.1</td>
<td>30.0</td>
<td>45</td>
</tr>
<tr>
<td>1/E(T) Rule</td>
<td>12.4</td>
<td>112.6</td>
<td>39.0</td>
<td>63</td>
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## Impact of Switching into a Life Annuity at Age 85 (US case)

### Results for Male (Retirement Age 65 Switching Age 75):

Benchmark Real Life Annuity €5.82 p.a./€100

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<td>55</td>
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Portfolio of Phased Withdrawal Plan and Deferred Life Annuity starting at Age 85 (US-Case)

Results for Male (Retirement Age 65 Switching Age 75):
Benchmark Real Life Annuity €5.82 p.a./€100

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<tr>
<td>Asset Class</td>
<td>Mean (% p.a.)</td>
<td>Volatility (% p.a.)</td>
<td>Correlations</td>
<td></td>
</tr>
<tr>
<td>-------------</td>
<td>---------------</td>
<td>---------------------</td>
<td>--------------</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Stocks</td>
<td>Bonds</td>
</tr>
<tr>
<td>Stocks</td>
<td>5.31 (5.53)</td>
<td>17.22 (25.36)</td>
<td>1</td>
<td>(0.235)</td>
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<tr>
<td>Bonds</td>
<td>3.31 (3.98)</td>
<td>11.78 (5.21)</td>
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<tr>
<td>Cash</td>
<td>1.41 (2.84)</td>
<td>2.35 (1.69)</td>
<td>0.446</td>
<td>0.591</td>
</tr>
</tbody>
</table>

Parenthesis: Results for German Capital Market
Literature