

Optimal Purchase of Life and Longevity Risk Insurance Products for Retired Couples

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Motivation and Research Question

Uncertainty of lifetimes is major risk for retired couples:

- risk of outliving their assets and leaving too little bequest
- risk of losing the income of the deceased spouse

How can this risk be hedged by the dynamic allocation using

- term life insurance
- single annuities
- joint and survivor annuities
- stocks and bonds



combinations

Literature and Contributions

Brown and Poterba (2000, JRI): welfare gains of full annuitization in joint and survivor annuities for couples in retirement

Love (2010, RFS): dynamic life cycle portfolio choice for families - investment universe: stocks, bonds, life insurance

Horneff et al. (2008, JRI): dynamic annuitization and portfolio choice in retirement framework

Our contribution: discrete time portfolio choice model for a couple in retirement with dynamic annuitization and life insurance purchases

Our Model – Preferences

Family Preferences

- utility is gained from consumption and bequest in CRRA framework (RRA $\gamma = 5$; time pref. $\beta = 0.96$)
- consumption is normalized by consumption scaling factor ϕ_s (“effective family size”)
→ couple: $\phi_s = 1.3$ singles: $\phi_s = 1$
- bequest Parameter $B = 2$ gives the strength of the bequest motive

$$J_t = \max\{u(C, s) + \beta E_t [J_{t+1}]\}$$

Markov chain

$$u(C, s) = \frac{1}{1 - \gamma} \left(\frac{C}{\phi_s} \right)^{1 - \gamma}$$

$$\text{Bequest} = \frac{1}{1 - \gamma} \left(\frac{W_t}{B} \right)^{1 - \gamma}$$

Our Model – Financial & Insurance Products

Financial and Insurance Products

- liquid wealth can be invested in

- riskless bonds (interest rate: $R_f - 1 = 2\%$)
- risky stocks (risk premium 4%, volatility 15.7%)

- renewable one-year term life insurance for each spouse

$$LP_t = L \cdot \frac{(1 - p_t)}{R_f}$$

price *face-value*

- single annuities for each spouse

$$AP_t = A \cdot \underbrace{\sum_{\tau=t+1}^T \frac{p_{\tau,t}}{(R_f)^{\tau-t}}}_{\ddot{a}_t}$$

price *payments* *annuity factor*

- joint annuities – constant payments till the last spouse dies

$$p_{\tau,t}^j = p_{\tau,t}^f + p_{\tau,t}^m - p_{\tau,t}^f p_{\tau,t}^m$$

Our Model – Policies

Decision Variables in each Period:

- consumption
- expenditures on life insurances
 - wife
 - husband
- expenditures on annuities
 - wife
 - husband
 - joint

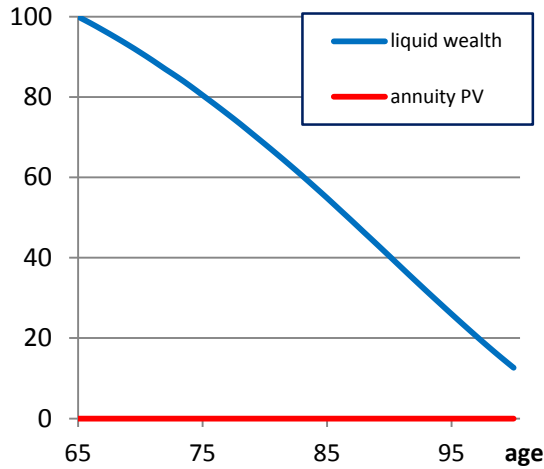
(availability is restricted to maximum age)
- allocation of (remaining) liquid wealth to stocks and bonds

Solution for optimal decisions found by backward induction of the value function.

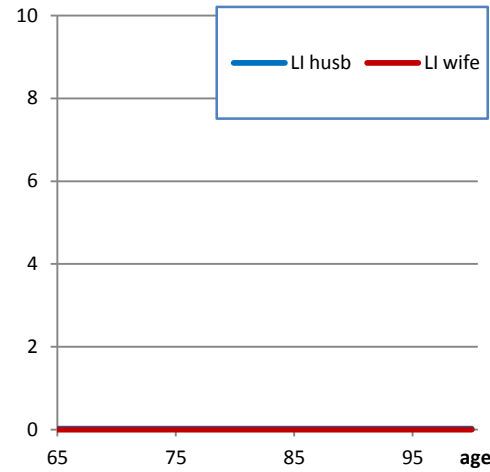
Life cycle profiles are means of 10,000 MC simulations conditioned on both spouses surviving using optimal decisions.

Life Cycle Profile without Annuities with and without Pre-Annuitized Wealth

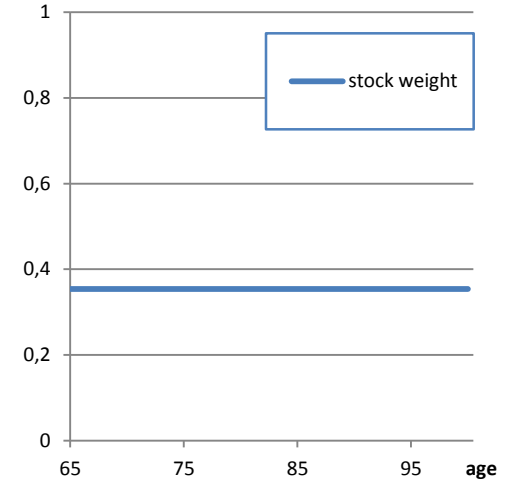
Wealth



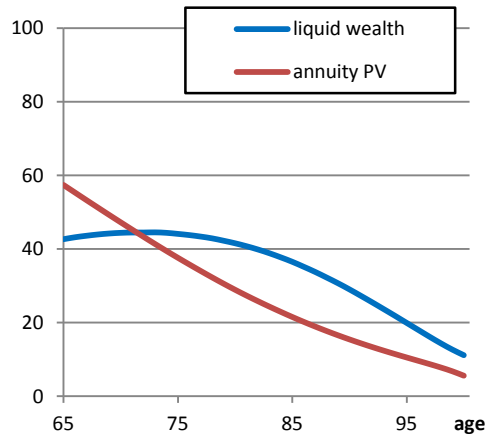
Life insurance (face values)



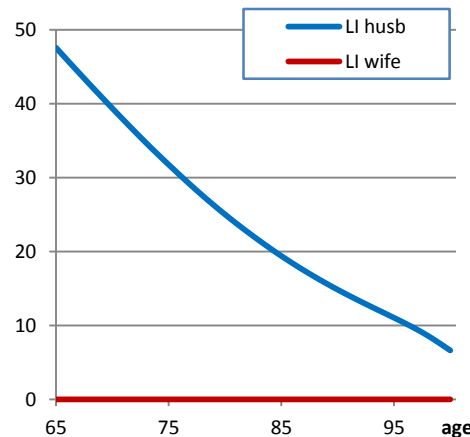
Stock weight



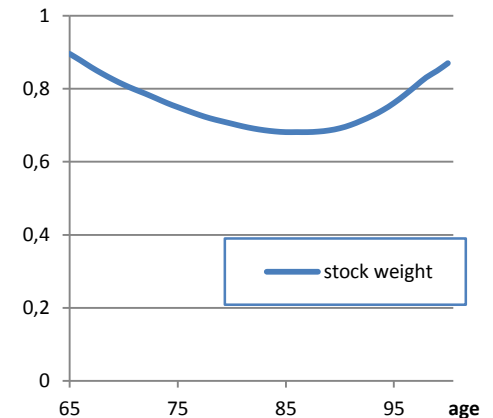
Wealth



Life insurance (face values)

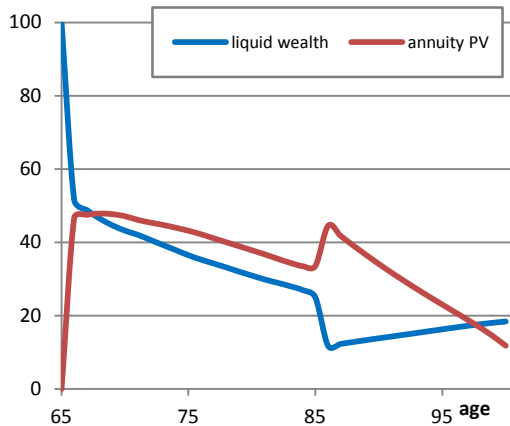


Stock weight

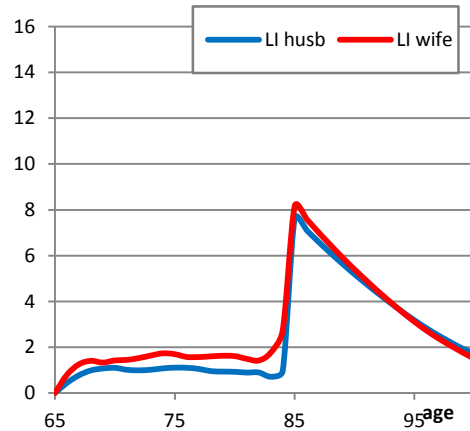


Life Cycle Profile with Single Annuities with and without Pre-Annuitized Wealth

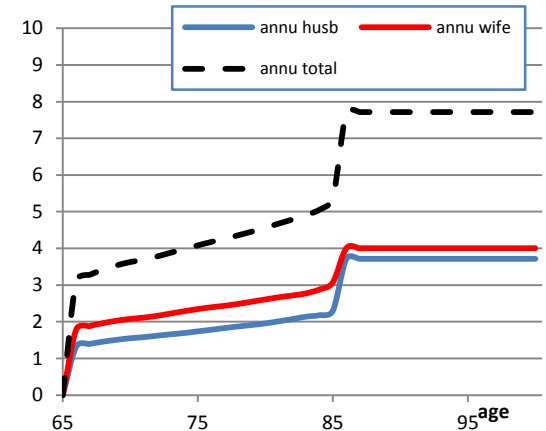
Wealth



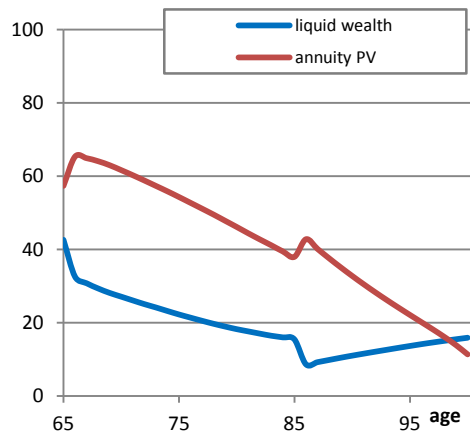
Life insurance (face values)



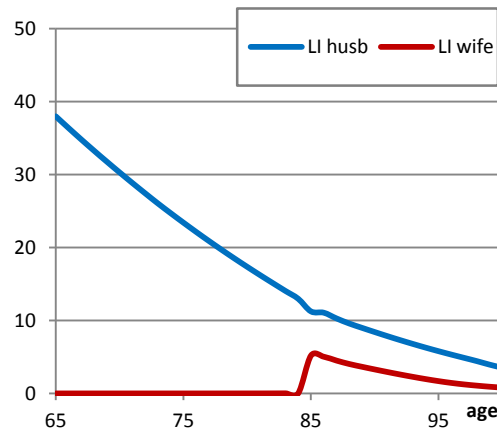
Annuity payments



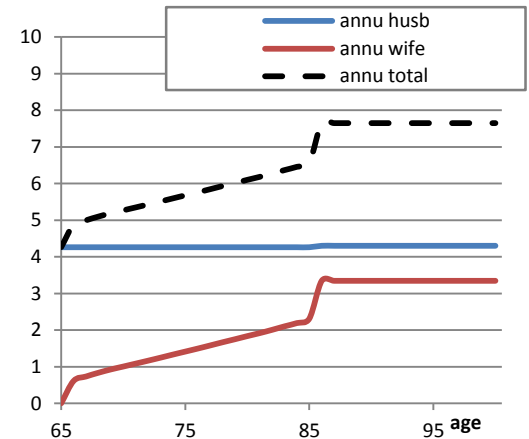
Wealth



Life insurance (face values)



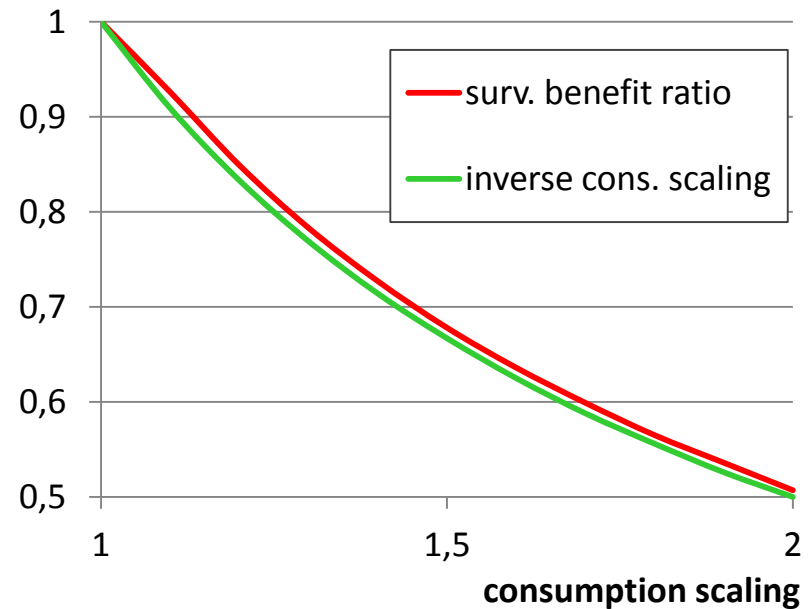
Annuity payments



Effect of Consumption Scaling on Survivor Benefit Ratio

- Household may purchase annuities only at the beginning of retirement.
- More than 70% of initial wealth is spent on a $K\%$ -joint annuity.
- The optimal survivor benefit ratio depends on the consumption scaling:

consumption scaling factor	survivor benefit ratio K
1.00	1.00
1.10	0.926
1.20	0.849
1.30	0.783
1.40	0.727
1.50	0.678
1.60	0.636
1.70	0.599
1.80	0.565
1.90	0.536
2.00	0.507



Sensitivity Analysis

- loadings (10%) on annuities and life insurance
 - annuitization is deferred to older ages (80-85), especially joint annuities
 - lower annuity-to-liquid-wealth ratios
- lower risk aversion ($\gamma = 2$)
 - annuitization is deferred (to gain equity premium on liquid wealth)
 - lower annuity-to-liquid-wealth ratio
 - higher risk aversion is vice versa (early annuitization, higher ratio)
- bequest strength
 - primarily influences the amount of liquid wealth at ages >85
 - absolutely no bequest → early annuitization
- increasing age difference (→ increasing difference in mortality)
 - more annuities for the wife
 - less joint annuities

Welfare Analysis

- certainty equivalent
at age 65:

$$CE_{65} = ((1 - \gamma) \cdot J_{65})^{\frac{1}{1-\gamma}}$$

	no pre-annuitization	with pre-annuitization (only husband)
annuities available up to age 85	1.00	0.98
no annuities available at all	0.83	0.85
annuities available only at age 65	0.99	0.94
annuities available up to age 85 no life insurance	0.999	0.86

Conclusion

- Joint & survivor annuities are useful products to hedge both kinds of mortality risk.
- Liquid wealth (invested mainly in stocks) is preferred over life insurance for bequest.
- Life insurance is used to insure pre-annuitized retirement wealth (e.g. DB pensions) of one spouse. Then they yield high welfare gains.

Thank
you !