How Efficient is the Individual Annuity Market?

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- The Single Premium Immediate Annuity (SPIA) market is becoming increasingly important within the financial economics literature.
- Most papers assume that this market is *efficient* and that – over short periods of time – prices respond to changes in interest rates only.
- We question that assumption.

[1.] **Portfolio Choice and Timing of Annuitization**
Research Question: *How much of personal retirement wealth should be allocated to life annuities and at what age should they be purchased?*

- Yaari (RES, 1965),
- Brugiavini (JPubE, 1993)
- Gerrard, Haberman and Vigna (IME, 2004), Kingston & Thorpe (JPEF, 2005), Stabile (IJTAF, 2006)
- Milevsky, Moore and Young (MathFin, 2006), Milevsky and Young (JEDC, 2007)
- Horneff, Maurer, Stamos (JEDC, 2008)
- Horneff, Maurer, Mitchell and Stamos (JPEF, 2010)
- Koijen, Nijman and Werker (RF, 2010)

[2.] **Money's Worth Ratio (MWR) Studies**

Research Question: *Do private-market life annuities provide "good value" around the world and how large is the adverse selection problem in pricing?*

- Warshawsky (JRI, 1988)
- Finkelstein and Poterba (JPE, 2004)
- Mitchell, Poterba, Warshawsky and Brown (AER, 1999)
- Cannon and Tonks (FHR, 2004)
- Fong, Lemaire and Tse (WP, 2011)

[3.] **Annuity Puzzle and Solutions**

Research Question: *If annuities are so great, then why don't more people voluntarily purchase annuities, and what can we do about it?*

- Yaari (RES, 1965), Davidoff, Brown and Diamond (AER, 2005)
- Dushi and Webb (JPubE, 2004)
- Lopez and Michaelidis (FRL, 2007)
- Butler and Teppa (JPubE, 2007)
- Pashchenko (WP, 2010)
- Inkman, Lopez and Michaelidis (RFS, 2011)
- Benartzi, Previtero and Thaler (JEP, 2011)
- Ameriks, et. al. (JF, 2011)

[4.] **Proper Valuation of Pension Liabilities**

Research Question: *A Defined Benefit (DB) pension plan creates deferred annuity-like liability for the sponsor and should be valued as such. Are sponsors doing this properly?*

- Treynor (JF, 1976), Bulow (QJE, 1982)
- Bodie (FAJ, 1990), Ippolito (FAJ, 2002)
- Sundaresan and Zapatero (RFS, 1997)
- Brown and Wilcox (AER, 2009)
- Novy-Marx and Rauh (JF, 2011)

All of these research strands implicitly or explicitly assume that

- private-market annuity prices quickly respond to changes in interest rates, and
- over short periods of time changes in annuity prices are primarily explained by changes in interest rates, and
- one can properly model the evolution of private-market annuity prices.

We believe our results are relevant (or at least should be of interest) to these audiences.

The common framework is the *No Arbitrage* Annuity Factor (AF):

$$\bar{a}(x, g, r) = \int_0^\infty e^{-rt} p(x, t) dt. \tag{1}$$

The (pseudo) survival probability is defined in the following way:

$$p(x, t) = \begin{cases} 1 & t \leq g \\ e^{-\int_0^t \lambda(x+s) ds} & t > g. \end{cases} \tag{2}$$

Using this notation, g is the guarantee (certain) period and $\lambda(x + s)$ is any general continuous mortality rate.

This implies that:

$$D_a := \frac{-\frac{\partial \bar{a}(x,g,r)}{\partial r}}{\bar{a}(x,g,r)} = \frac{1}{r} - \frac{\bar{A}^t(x,g,r)}{r\bar{a}(x,g,r)}, \tag{3}$$

where $\bar{A}^t(x,g,r)$ is the No Arbitrage *net single premium* (NSP) of a deferred life insurance policy that pays t dollars upon death, instead of the standard \$1. So, if the insured dies in year $t = 10 > g$, the beneficiaries get \$10, etc.

The *theoretical* duration of the life annuity factor is:

$$D(x,g,r) := \frac{-\frac{\partial \bar{a}(x,g,r)}{\partial r}}{\bar{a}(x,g,r)} \approx \frac{-\triangle \bar{a}(x,g,r)/\triangle r}{\bar{a}(x,g,r)}. \tag{4}$$

So, empirically, if the pair $(\tilde{a}_i, \tilde{r}_i)$ denotes the *observed* annuity factor and corresponding interest rate at time period i, and $(\tilde{a}_{i+1}, \tilde{r}_{i+1})$ denotes the same pair in the next period, then:

$$\frac{-\left(\tilde{a}_{i+1} - \tilde{a}_i\right)}{\tilde{a}_i} = \frac{-\triangle \tilde{a}_i}{\tilde{a}_i} = \tilde{D}(r_{i+1} - r_i) = \tilde{D} \triangle r_i, \qquad (5)$$

where D is the *empirical* duration. So, if we define $y := \frac{-\triangle \tilde{a}_i}{\tilde{a}_i}$ (dependent variable) and $x = r_{i+1} - r_i$ (independent variable) and run the regression:

$$y_i = d_0 + d_1 x_i + e_i, \qquad (6)$$

we would expect that $d_0 = 0$ and $d_1 = D(x, g, r)$.

- Assume that $r = 4.35\%$ and the mortality rates $\lambda(t)$ obeys a Gompertz law of mortality with $m = 92.63$ and $b = 8.78$. This implies that $p(65, 35) = 10.3\%$, which is the survival probability from the Individual Annuity Mortality table. (Source: Milevsky and Young, 2007).
- Here are duration values based on the model:

$$\begin{bmatrix} g = 10 \text{ (years)} & a(x,g,r) & \frac{\partial \bar{a}(x,g,r)}{\partial r} & D_a \text{ (Duration)} \\ \text{Age } x = 55 & \$17.02 & -220 & 12.9 \\ \text{Age } x = 65 & \$14.44 & -151 & 10.4 \\ \text{Age } x = 76 & \$11.51 & -90 & 7.8 \end{bmatrix}$$

- Main claim: *Empirically all of our numbers are much smaller than this.*

The Data

- Our dataset is compiled by QWeMA/CANNEX, which offers annuity payout rates per $100,000 premium, for all commonly offered life annuity policies in the United States.
- The database contains over three million individual quotes spanning seven years and twenty five life insurers.
- Quotes are classified by age; gender, guarantee period; mortality dependence, such as single life, joint life, and term certain; and qualified vs. non-qualified status.
- The database is updated weekly, and is validated and scrubbed for irregularities.

- We focused on qualified (i.e. purchased with funds within IRA or 401K accounts) and averaged the payout rates (minus outliers) across all U.S. insurance companies quoting on a given date.
- We then focused on the following six age groups 55, 60, 65, 70, 75, 80 and five guarantee periods 0, 5, 10, 15, 20.
- In total, each observation date consisted of a matrix of 30 numbers for males and 30 numbers for females.
- The monthly annuity payouts (MAP) were converted into annuity factors (AF) by annualizing the monthly payout and then dividing into the $100,000 premium.

Summary Statistics: Monthly Annuity Payouts

Term	N	Mean	Std	Min	Max	Skew	Kurt
0	296	544	31	458	599	-0.90	0.31
5	300	542	31	457	596	-0.90	0.23
10	302	536	30	454	590	-0.84	0.14
15	303	528	30	446	581	-0.85	0.17
20	299	518	29	438	570	-0.83	0.20

Notes: Males, Qualified, Age 55, date range: 2004-2010

Summary Statistics: Monthly Annuity Payouts

Term	N	Mean	Std	Min	Max	Skew	Kurt
0	296	875	44	756	939	-1.21	0.58
5	296	845	40	734	904	-1.21	0.68
10	298	770	35	674	824	-1.08	0.48
15	267	692	38	598	735	-0.98	-0.10
20	271	618	41	524	670	-0.75	-0.55

Notes: Males, Qualified, Age 75, date range: 2005-2010

Summary Statistics: Monthly Annuity Payouts

Term	N	Mean	Std	Min	Max	Skew	Kurt
0	304	518	30	440	574	-0.70	-0.06
5	298	518	30	436	572	-0.76	0.10
10	298	514	29	434	568	-0.73	0.07
15	299	509	29	430	563	-0.73	0.06
20	299	503	29	424	556	-0.73	0.06

Notes: Females, Qualified, Age 55, date range: 2004-2010.

Summary Statistics: Monthly Annuity Payouts

Term	N	Mean	Std	Min	Max	Skew	Kurt
0	296	779	36	680	841	-0.91	0.30
5	296	733	33	642	789	-0.91	0.31
10	296	675	33	586	720	-1.02	0.31
15	266	612	37	520	657	-0.92	-0.18

Notes: Females, Qualified, Age 75, date range: 2005-2010

Summary Statistics: Select Δ Annuity Factors and Rates

Variable	N	Mean	Std	Min	Max	Skew	Kurt
Δ 10 Y Rate	332	0	.001	-.007	.005	-.16	2.5
Δ AF M, Q, Age 55, Term 0	277	0	.099	-0.5	.45	-.3	4.3
Δ AF F, Q, Age 55, Term 0	287	0	.11	-.51	.50	-.3	3.9

Notes: Date range: 2005-2010. Term 0 is a 0 year guarantee period.

Gender: M Qualified Status: Q

$$\frac{\partial \bar{a}(x,g,r)}{\partial r}$$

Age	Guarantee Period 20	15	10	5	0
55	**-19.3**	**-20.4**	**-19.4**	**-20.2**	**-19.0**
60	**-17.5**	**-16.2**	**-14.4**	**-15.9**	**-15.5**
65	**-19.0**	**-13.9**	**-12.1**	**-12.7**	**-12.3**
70	**-4.62**	**-11.0**	**-9.64**	**-9.47**	**-8.68**
75		**-2.19**	**-7.44**	**-6.85**	**-5.92**
80			**0.75**	**-4.48**	**-3.80**

Notes: **Bolded coefficients** denote significant difference from theoretical value at the 5 percent level, based on two-sided tests. One-at-a-time Equation Estimation.

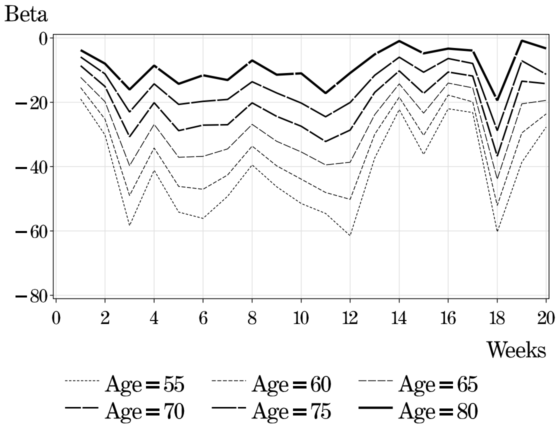
Gender: M Qualified Status: Q

D_a **(Duration)**

	Guarantee Period				
Age	20	15	10	5	0
55	**1.20**	**1.28**	**1.24**	**1.31**	**1.24**
60	**1.14**	**1.09**	**1.00**	**1.12**	**1.09**
65	**1.28**	**1.00**	**0.92**	**0.99**	**0.96**
70	**0.31**	**0.85**	**0.80**	**0.83**	**0.77**
75		**0.17**	**0.68**	**0.68**	**0.61**
80			**0.10**	**0.52**	**0.48**

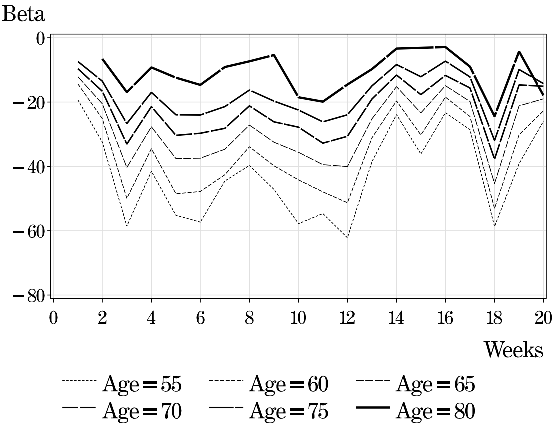
Notes: **Bolded coefficients** denote significant difference from theoretical value at the 5 percent level, based on two-sided tests. One-at-a-time Equation Estimation.

$\frac{\partial \bar{a}(x,g,r)}{\partial r}$ Males, Qualified, 0 Year Guarantee.



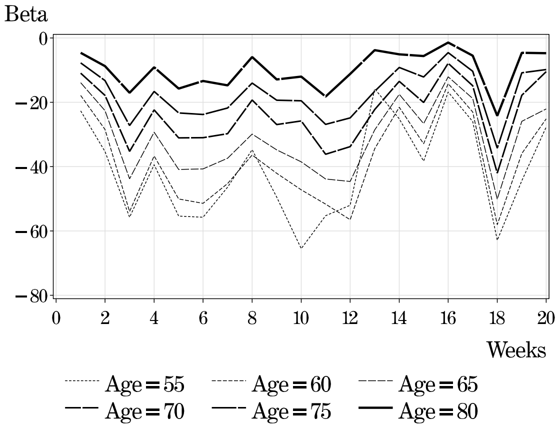
$$\Delta a_t(term = 0|Weeks)/\Delta r_t(10\ year\ swap|Weeks)$$

$\frac{\partial \bar{a}(x,g,r)}{\partial r}$ Males, Qualified, 10 Year Guarantee.



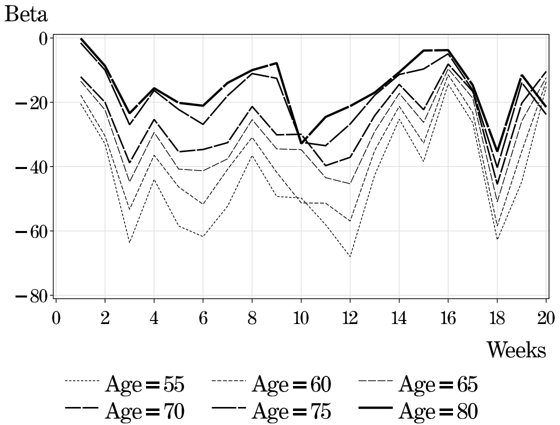
$$\Delta a_t(term = 10|Weeks)/\Delta r_t(10\ year\ swap|Weeks)$$

$\frac{\partial \bar{a}(x,g,r)}{\partial r}$ Females, Qualified, 0 Year Guarantee.



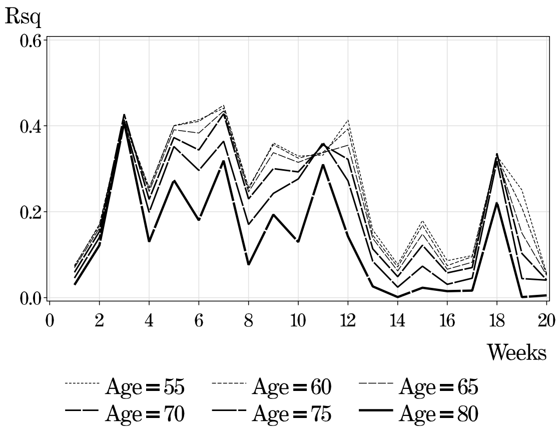
$$\Delta a_t(term = 0|Weeks)/\Delta r_t(10\ year\ swap|Weeks)$$

$\frac{\partial \bar{a}(x,g,r)}{\partial r}$ Females, Qualified, 10 Year Guarantee.



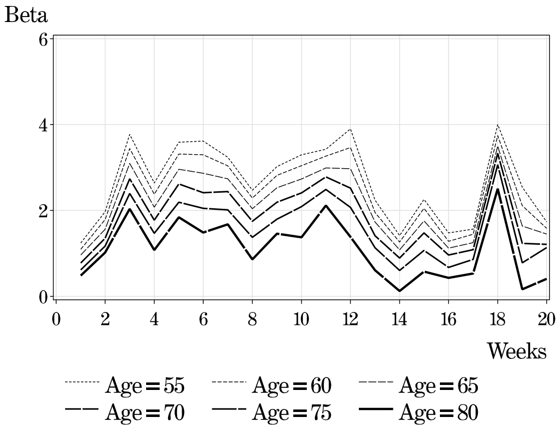
$$\Delta a_t(term = 10|Weeks)/\Delta r_t(10\ year\ swap|Weeks)$$

R^2 of Model for Males, Qualified, 0 Year Guarantee.



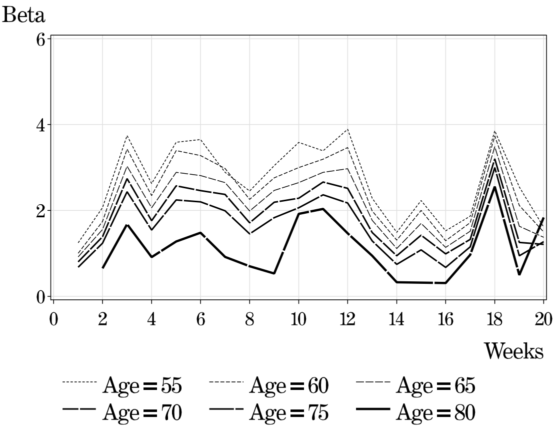
$$\Delta a_t(term = 0|Weeks)/\Delta r_t(10\ year\ swap|Weeks)$$

D_a **(Duration)** Males, Qualified, 0 Year Guarantee.



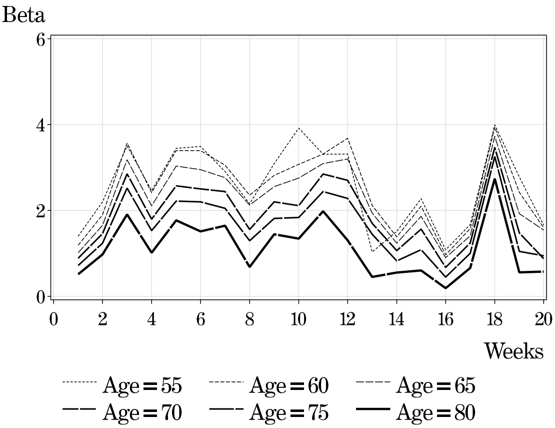
$$D(x,g,r) := \frac{-\frac{\partial \bar{a}(x,g,r)}{\partial r}}{\bar{a}(x,g,r)} \approx \frac{-\triangle \bar{a}(x,g,r)/\triangle r}{\bar{a}(x,g,r)}. \quad (7)$$

D_a **(Duration)** Males, Qualified, 10 Year Guarantee.



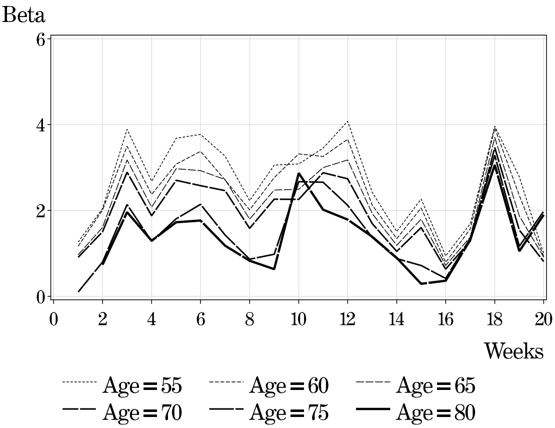
$$D(x,g,r) := \frac{-\frac{\partial \bar{a}(x,g,r)}{\partial r}}{\bar{a}(x,g,r)} \approx \frac{-\triangle \bar{a}(x,g,r)/\triangle r}{\bar{a}(x,g,r)}. \quad (8)$$

D_a (Duration) Females, Qualified, 0 Year Guarantee.



$$D(x,g,r) := \frac{-\frac{\partial \bar{a}(x,g,r)}{\partial r}}{\bar{a}(x,g,r)} \approx \frac{-\triangle \bar{a}(x,g,r)/\triangle r}{\bar{a}(x,g,r)}. \quad (9)$$

D_a **(Duration)** Females, Qualified, 10 Year Guarantee.



$$D(x,g,r) := \frac{-\frac{\partial \bar{a}(x,g,r)}{\partial r}}{\bar{a}(x,g,r)} \approx \frac{-\triangle \bar{a}(x,g,r)/\triangle r}{\bar{a}(x,g,r)}. \tag{10}$$

Conclusions

- Annuity prices do not respond to changes in interest rate in a way one might expect from (theoretical) models used in the portfolio choice and life-cycle literature.
    - Duration values are much smaller in magnitude than predicted.
    - Life annuity prices take 3-7 weeks to respond to changes in interest rates.
    - Little variability in prices is explained.
    - Price fluctuations are very exaggerated relative to interest rate fluctuations.
    - (Unreported) annuity prices are more likely to change when interest decline as opposed to increase.
- Overall, our results suggest substantial market timing benefits and raise concerns about money worth ratio (WMR) studies.