ROTH VERSUS TRADITIONAL ACCOUNTS IN A LIFE-CYCLE MODEL WITH TAX RISK

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## FRONT- VS. BACK-LOADED RETIREMENT ACCOUNTS

<table>
<thead>
<tr>
<th></th>
<th>Contributions tax deductible?</th>
<th>Investment income taxable?</th>
<th>Withdrawals taxable?</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Front-loaded</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>RRSPs</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Traditional IRA/401(k)s</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Back-loaded</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>TFSAs</td>
<td>No</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Roth IRA/401(k)s</td>
<td>No</td>
<td>No</td>
<td>No</td>
</tr>
</tbody>
</table>
WHICH IS BETTER?

- **Standard argument**: (e.g. Engen et al., 1994)

  \[ \tau^{br} = \text{marginal tax rate before retirement} \]
  \[ \tau^{ar} = \text{marginal tax rate after retirement} \]

  \[ \tau^{br} > \tau^{ar} \rightarrow \text{RRSP/traditional (front-loaded) is better} \]
  \[ \tau^{br} = \tau^{ar} \rightarrow \text{Indifferent} \]
  \[ \tau^{br} < \tau^{ar} \rightarrow \text{TFSA/Roth (back-loaded) is better} \]

Note: This comparison ignores other considerations such as tax penalties, taxation to heirs, and required minimum distributions.
Tax diversification?

Vanguard: “In a world of uncertain future tax rates, participants should diversify. Just as they hold fixed income assets to diversify the risks of stocks, so participants should hold Roth savings to diversify the risks associated with pre-tax savings.”
## MOTIVATION

<table>
<thead>
<tr>
<th><strong>Conventional view</strong></th>
<th><strong>Not so simple, tax rates are:</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>• Tax rates should decrease with income after retirement</td>
<td>• Risky</td>
</tr>
<tr>
<td>• Front-loaded accounts should be better in all but rare cases (Butterfield et al. 2000)</td>
<td>• Lower in early career</td>
</tr>
<tr>
<td></td>
<td>• Endogenous</td>
</tr>
<tr>
<td></td>
<td>• Bracketed</td>
</tr>
<tr>
<td></td>
<td>• Effectively much higher when withdrawals affect SS benefits or their taxation (Kotlikoff et al., 2008)</td>
</tr>
</tbody>
</table>
TAXATION OF U.S. SOCIAL SECURITY BENEFITS

• Provisional Income (PI) = ½ SS+ income
  • Includes traditional withdrawals, not Roth

• Taxable SS benefits (for singles) are:
  • 50% of PI between $25,000-$34,000
  • 85% of PI over $34,000
  • Maximum taxable: 85% of SS benefits

• Example:
  • SS=$15,000; Withdrawal=$20,000; τ=15%
  • SS_{\text{Taxable}}=50\% \times (1/2 \times $15,000 + $20,000-$25,000)
  • Effective $\tau$ on withdrawal: 15%+50\% \times 15\% = 22.5%
EFFECTIVE TAX RATES:
Can be higher after retirement without having higher income

Example: Taxable income before retirement: $100,000
Taxable income after retirement: $80,000

After retirement, depending on SS taxation, effective tax rate can be either 25%, 38%, or 46%
15% of OAS benefits repaid for income > $69,562

**Example:**
- Taxable income before retirement: $100,000
- Taxable income after retirement: $80,000

After retirement, tax rate is 37%
Before retirement, tax rate is 26%

Federal tax rates vs. With 15% OAS repayment
MODEL
LIFE-CYCLE MODEL

Starting point: Yaari (1965)

Add back-loaded accounts
- Solution same as without taxes

Add front-loaded accounts with tax brackets
- Discontinuity issues when tax rates jump
- Solution for $c^*$ takes two forms

Add Social Security taxation
- Unusual structure creates potential for jump in optimal consumption process

Add tax risk: Not that bad, mostly longer
MODELING TAXES

• $\tau_k$: Marginal tax rate in tax bracket $k=1,...,K$
• $\tau_k(1 + M_h)$: Effective marginal tax rate, where $M_h$ is 0%, 50%, or 85%
• Tax risk:
  • One-time increase $\theta$ after retirement: $\tilde{\tau}_k = \tilde{\theta}\tau_k$
  • $\tilde{\theta} = \theta_i$ with probability $p_i$ for $i = 1, ..., N$
• $\alpha$: Fixed proportion of savings/withdrawals allocated to the front-loaded account.
• $s$: savings
• Taxable income adjusted by $-\alpha \cdot s$
OPTIMIZATION PROBLEM

\[
\max_c E \left[ \int_{t_0}^{\omega} \frac{f(t)}{e^{-\beta(t-t_0)p_{t_0,t}}} \cdot \frac{u(c_t)}{c^{1-\gamma}} \frac{1}{1-\gamma} dt \right]
\]

s.t.
\[
dW_t = [W_t r + y_t - c_t - tax(s_t(c_t))] dt
\]

\[W_t \geq 0, \quad W_{t_0} = 0.\]
ANALYTICAL SOLUTION WITH DUAL APPROACH

1. Write Lagrangian: \( (X(t)) \) is a non-increasing process equal to a constant \( \lambda \) when \( W_t > 0 \) and a given function \( \lambda(t) \) when \( W_t = 0 \)

\[
L = E \left[ \int_t^\infty [f(t)u(c_t)dt + X(t)e^{-r(t-t_0)}s(c_t)]dt \right]
\]

2. Find \( c^* \) that maximizes \( L \)

3. Instead of showing that \( W_t \geq 0 \) for all \( t \), find \( X(t) \) that satisfies the budget constraint and minimizes \( L(c^*) \).
F.O.C. $L'(c) = 0$: DISCONTINUITY ISSUES

- $L'(c) = f(t)u'(c) - \lambda e^{-r(t-t_0)}/(1 - \alpha \cdot \tau_k(1 + M_h))$

**Continuous tax rates**

- With $L''(c) < 0$, works well, solution to $L'(c) = 0$ exists and is unique

**Discontinuous tax rates**

- **Tax brackets**: make $L'(c)$ jump down, may not have a solution to $L'(c) = 0$
- **Reaching SS taxable max**: makes $L'(c)$ jump up, may have two solutions to $L'(c) = 0$
We have two possible local optimums: one before the SS taxable maximum is reached and one after.
ANALYTICAL SOLUTION

• Solution when $W_t > 0$ has two possible forms:

\[ c^* = \begin{cases} 
\left( \frac{\lambda e^{-r(t-t_0)}}{f(t)(1 - \alpha \tau_k (1 + M_h))} \right)^{-\frac{1}{\gamma}} & \text{Interior solution} \\
C_k & \text{Corner solution} 
\end{cases} \]

• Without tax risk: $\lambda$ solves the budget constraint.

• With tax risk: $\lambda$ becomes $\lambda_i$ in state $i$ after retirement. We have $N$ budget constraints equations and an additional condition $\lambda = \sum_{i=1}^{N} \lambda_i p_i$. 
NUMERICAL ILLUSTRATIONS: WITHOUT TAX RISK
EXAMPLE:
NO HIGH SCHOOL, NO PENSION

Marginal tax rate 15%
Effective marginal tax rates are the same in this case

Marginal tax rate

After-tax income

Age 45 in 2010
EXAMPLE: HIGH SCHOOL, HIGH PENSION

Marginal tax rate
- 15%

Effective marginal tax rate
- 27.75%
- 18.5%
- 10%

After-tax income
- Roth
- Traditional

Age 25 in 2010
EXAMPLE:
COLLEGE, NO PENSION

Marginal tax rate
25%

Effective marginal
tax rates
25% 25% 27.75%
15% 18.5% 10%

After-tax income
–
c* Roth
c* Traditional

Age 25 in 2010
WELFARE GAINS: TRAD. VS. ROTH (BY PENSION LEVEL)

<table>
<thead>
<tr>
<th>Education Level</th>
<th>No Pension</th>
<th>Medium Pension</th>
<th>High Pension</th>
</tr>
</thead>
<tbody>
<tr>
<td>No High School</td>
<td>$14,184</td>
<td>$10,250</td>
<td>$6,677</td>
</tr>
<tr>
<td>High School</td>
<td>$17,114</td>
<td>$9,357</td>
<td>$2,132</td>
</tr>
<tr>
<td>College</td>
<td>$26,461</td>
<td>($393)</td>
<td>($4,388)</td>
</tr>
</tbody>
</table>

Age 45 in 2010
TAX RISK
WELFARE GAINS WITH STRATEGIES MIXING ACCOUNTS

Mixed strategies:
Can be optimal even without tax risk to avoid triggering higher taxes

Tax diversification:
(with 50%/50% strategy)
Risk-reduction gain: $281
Expected return loss: $2,268

Fraction invested in traditional account ($\alpha$)

<table>
<thead>
<tr>
<th>Fraction</th>
<th>No high school</th>
<th>High school</th>
<th>College</th>
<th>With tax risk</th>
</tr>
</thead>
<tbody>
<tr>
<td>0%</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>20%</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>40%</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>60%</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>80%</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>100%</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Welfare Gain/Loss ($): $2,000, $4,000, $6,000, $8,000, $10,000, $12,000

Tax risk in illustration: 50%/50% chance of tax rates going up or down by 20%
CONCLUSION

Retirement planning → Complex
+
Future tax rules → Complex
=
Front-loaded acct. incentives → Complex^2